

A Comprehensive Review of Recent Variants and Modifications of Firefly Algorithm

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ABSTRACT

Swarm intelligence (SI) is an emerging field of biologically-inspired artificial intelligence based on the behavioral models of social insects such as ants, bees, wasps, termites etc. Swarm intelligence is the discipline that deals with natural and artificial systems composed of many individuals that coordinate using decentralized control and self-organization. Most SI algorithms have been developed to address stationary optimization problems and hence, they can converge on the (near-) optimum solution efficiently. However, many real-world problems have a dynamic environment that changes over time. In the last two decades, there has been a growing interest of addressing Dynamic Optimization Problems using SI algorithms due to their adaptation capabilities. This paper presents a broad review on two SI algorithms: 1) Firefly Algorithm (FA) 2) Flower Pollination Algorithm (FPA). FA is inspired from bioluminescence characteristic of fireflies. FPA is inspired from the the pollination behavior of flowering plants. This article aims to give a detailed analysis of different variants of FA and FPA developed by parameter adaptations, modification, hybridization as on date. This paper also addresses the applications of these algorithms in various fields. In addition, literatures found that most of the cases that used FA and FPA technique have outperformed compare to other metaheuristic algorithms.

Keywords-- Firefly Algorithm, Flower Pollination Algorithm, Swarm Intelligence, Hybridizations, Parameter Tuning

I. INTRODUCTION

Special characteristic of many natural phenomenon can be converted to an algorithm or mathematical models to solve real world complex problems. Many natural phenomena have been converted to algorithms by researchers to find optimal solutions of optimization problems. These include genetic algorithm (GA), particle swarm optimization (PSO), firefly algorithm (FA), flower pollination algorithm (FPA), cuckoo search (CS), artificial bee colony (ABC), grey wolf algorithm (GWO) and bat algorithm (BA). These algorithms have been applied to many optimization

problems for last some decades. FA is one of the recent swarm intelligence methods developed by Yang in 2008. This algorithm belongs to stochastic algorithms. This means that it uses a kind of randomization by searching for a set of solutions. It is inspired by the flashing lights of fireflies in nature. Heuristic means 'to find' or 'to discover solutions by trial and error'. Flower pollination algorithm (FPA) describing the behaviour of flower reproduction has been proposed by Yang (2012). FPA is a swarm-based optimization technique that has attracted the attention of many researchers in several optimization fields due to its impressive characteristics. FPA has very fewer parameters and has shown a robust performance when applied in various optimization problems. In addition, FPA is a flexible, adaptable, scalable, and simple optimization method. Therefore, FPA, compared with other metaheuristic algorithms, shows good results for solving various real-life optimization problems from different domains.

Each meta-heuristic search process depends on balancing between two major components: exploration and exploitation. For the meta-heuristic algorithms [19], the exploration denotes the process of discovering the diverse solutions within the search space, while exploitation means focusing the search process within the vicinities of the best solutions, thus, exploiting the information discovered so far.

Therefore, this article is organized as follows. Section 2 describes the theory and variants of Firefly algorithm (FA); Section 3 describes the theory and variants of Flower pollination Algorithm (FPA). Some applications are briefly outlined in Sect. 4, and and critical analysis of FPA is carried out in Sect. 5. Finally, the conclusion will be drawn with some recommendation in Section 6

II. FIREFLY ALGORITHM

Firefly algorithm formulated by Yang (Yang and Karamanoglu 2013) is a metaheuristic algorithm that is inspired by the flashing behavior of fireflies and the phenomenon of bioluminescent communication. Yang

formulated the Firefly Algorithm with the following assumptions:

1) A firefly will be attracted to each other regardless of their sex because they are unisexual.

2) Attractiveness is proportional to their brightness whereas the less bright firefly will be attracted to the brighter firefly. However, the attractiveness decreased when the distance of the two fireflies increased.

3) If the brightness of both fireflies is the same, the fireflies will move randomly. The generations of new solutions are by random walk and attraction of the fireflies (Yang 2010b)

The brightness of the fireflies should be associated with the objective function of the related

problem. Their attractiveness makes them capable to subdivide themselves into smaller groups and each subgroup swarm around the local models. Thus, FA is suitable for optimization problems as stated by (Yang 2010a, b; Yang and Karamanoglu 2013)

These three rules may be used to create an optimization algorithm with the additional feature that the brightness is proportional to the value of the objective function. The firefly rules can then be turned into steps in the algorithm by generating the positions of an initial population of fireflies, calculating the value of the objective function for each of these fireflies, and then applying these rules for a number of generations. (Lindfield and Penny 2017). The following equation allows the updating of the positions.

$$x^{(t+1)} = x^{(t)} + \beta_0 e^{-\gamma d_{ij}^2} (x_i^{(t)} - x_j^{(t)}) + \alpha r_i \quad (1)$$

where t denotes the iteration or generation number of the process, d_{ij} is the distance between any of the pairs of fireflies i and j , r_i a vector chosen from a uniform or normal distribution and γ a user set parameter usually 1. This equation can be interpreted as including

$$\beta_0 e^{-\gamma d_{ij}^2} (x_i^{(t)} - x_j^{(t)}) \quad (2)$$

Provides control of the convergence of the fireflies towards each other and ultimately the convergence to a specific point. As the distance between the fireflies decreases the exponential term will approach 1. But when the distance between fireflies increases then this term decreases because the power in the exponential function becomes increasingly negative and the

two basic elements for the modification of the firefly position values. The term αr_i allows the random exploration of the region and the extent of its influence can be modified by adjusting the parameter α and the term:

exponential term becomes smaller. The overall effect of this term can be controlled by the constant β_0 . The value of r_i is selected from a random distribution which maybe a uniform distribution adjusted to be in the range $[-1, 1]$ or standard normal distribution maybe used.

The pseudo algorithm of FA (Gayathri Devi et al. 2022) is given in Table 1

Table 1: Pseudocode of Firefly Algorithm (FA)

Objective function $F(x)$, $x = (x_1, \dots, x_d)^T$
Generate initial population of fireflies $x_i = (1, 2, \dots, n)$
Light intensity I_i at x_i is ascertained by $f(x_i)$
State each parameter γ , G_{\max} , α , β_0
while ($t < G_{\max}$)
for $i = 1:n$ all n fireflies
for $j = 1:n$ all n fireflies
Attractiveness varies with distance r via $e^{-[\gamma r^2]}$
if ($X_j < X_i$) then (if firefly j is better)
Analyze new solutions and updation of light intensity
end for j
end for i
Rank the fireflies and find the current global best
End while
Postprocess results and visualization

III. VARIANTS AND MODIFICATIONS OF FA

In FA the parameters are fixed, the search behaviour remains to be the same for any condition in all

iterations. Hence modifying the standard firefly algorithm to boost its performance has been one of the research issues. Furthermore, the standard firefly algorithm is designed for continuous optimization problems; hence in order to use it for discrete problems it needs to be modified and adjusted.

The FA variants are based on

- Parameters
- Updating strategy
- Hybridizing with other metaheuristics

3.1 Modifications based on FA parameters

The firefly algorithms depend primarily on the variation of light intensity and the formulation of attractiveness. Both factors allow significant scope for algorithm improvements. Some of the modifications of firefly algorithm are done by making these parameters variable and adaptive. In recent researches on the modification of firefly algorithms, the parameters ' α ' (randomness parameter), ' γ ' (Light absorption Co-efficient), ' β ' (Attractiveness) and ' r ' (Distance) are modified. The modification of α affects the random moment of the firefly, whereas modifying

either γ or r affects the degree of attraction between fireflies. Adjusting the brightness at the origin, β_0 , has also been done in some researches. (Khan et al. 2016)

In standard FA the randomness parameter ' α ' remains fixed and does not change during iterations. The value of the step size factor α has a great influence on the searching ability of the algorithm. Through analysis and experiments, it is found that the value of α is larger, and the global optimization is better. But it is not easy to converge in the later period of the algorithm. The smaller value of α has better convergence but is easy to fall into the local optimum. Therefore, researchers started adapting the ' α ' to obtain better optima solutions.

(Fister et al. 2012) proposed a memetic FA (MFA), in which the parameter α is dynamically updated as the iteration changes. The new α is defined by

$$\alpha(t+1) = \left(\frac{1}{9000}\right)^{1/t} \alpha(t) \quad (3)$$

where t represents the iteration number or generation index.

(Yu et al. 2014) proposed a wise strategy for step setting (WSSFA), which considers the information of firefly's personal and the global best positions. The

authors calculated the step for each firefly separately, based on its best position at each iteration. The proposed strategy is presented mathematically by the following equation:

$$\alpha(t+1) = \alpha_i(t) - \alpha_{min} * e^{-(|x_{gbest}(t) - x_{ibest}(t)| * (t/max_iter))} \quad (4)$$

WSSFA was tested on 20 benchmark problems for minimization and maximization problems and the authors found the results to be effective.

(Yu et al. 2013) proposed a self-adaptive step FA (SASFA). In this paper, firefly's history information includes its past two iteration's optimum value. The authors developed two new equations for past two iterations' history information and the current information of fitness value. So, the step of each firefly can vary with the iteration, and the step of each firefly is also altered at the same iteration. Obviously, the step of each firefly is different for various problems, because different fitness functions are used. Based on the equations developed, the α step gets changed at every step.

(Yu et al. 2015) proposed variable step size FA (VSSFA). In this paper, the authors use a nonlinear equation and design a dynamic adjusting scheme of step. During iterations, the step is large at early stage, and then decreases with the iteration increases. This can help algorithm to balance the ability of global exploration and local exploitation. The authors tested it on sixteen benchmark functions with an objective to minimize.

(Liu et al. 2020) proposed a dynamic adaptive firefly algorithm with gaussian distribution (GDAFA). In this paper, the adaptive random step size mechanism based on the optimal deviation degree is introduced to dynamically adjust the step size factor according to the current optimal value, to improve the adaptability of the algorithm and enhance the population diversity at the same time. Comparison with six kinds of algorithms in 18

different characteristics test functions on the experimental results of contrast fully shows that the GDAFA optimization performance and convergence rate are significantly increased.

(Cheung et al. 2014) developed an adaptive firefly algorithm (AdaFa) by making adaptations on (1) a distance-based light absorption coefficient; (2) a gray coefficient enhancing fireflies to share difference information from attractive ones efficiently; and (3) five different dynamic strategies for the randomization parameter.

An adaptive FA is proposed by (Baykasoğlu and Özsoydan 2015) paper to solve mechanical design optimization problems. The adaptivity is focused on the search mechanism and adaptive parameter settings. chaotic maps are also embedded into AFA for performance improvement

(Wang et al. 2017) investigate the control parameters of FA, and propose a modified FA called FA with adaptive control parameters (ApFA). The authors designed two simple parameter strategies: adaptive α and dynamic β_0 . The first strategy will guarantee the mentioned prerequisite for the convergence of FA, and the second one can avoid the invariableness of the attractiveness by adjusting the β_0 .

In small-sized optimization problems with a small number of design variables, the neighbouring fireflies are close to each other and β gives a meaningful value. On the other hand, in high-dimensional and large-sized problems, the neighbouring fireflies are too far

away from each other and the value of β gets smaller. Therefore (Carbas 2016) proposed an Enhanced Firefly Algorithm (EFFA) by adapting attractiveness β and randomness parameter α . These parameters are changed dynamically during the design iterations by the equations developed by the author. EFFA demonstrated significant improvement in the design examples by improving the efficiency of the standard FA.

(Lieu et al. 2018) presented a novel adaptive hybrid evolutionary firefly algorithm (AHEFA) for shape and size optimization of truss structures under multiple frequency constraints. This algorithm is a hybridization of the differential evolution (DE) algorithm and the firefly algorithm (FA). An automatically adapted parameter computed from the deviation of objective function between the best individual and the whole population in the previous generation is used to select an appropriate mutation operator for the performance in the mutation phase.

(Sababha et al. 2018) introduces a modified exploration and exploitation mechanism, with adaptive randomness (α) and absorption coefficients (γ). In this paper the authors set the randomization and attractiveness parameter in advance at the stage of initialization and later modified in the process of optimization. They tested the modified algorithm on two types of optimization problems that form the functions are local optima and a single global optimum. They consist of multimodal and unimodal types.

(Ghasemi et al. 2022) proposed a novel FA, called firefly algorithm 1 to 3 (FA1→3), via different types of movements of fireflies in an attempt to improve the global exploration and convergence characteristics of FA. The CEC2014 test functions were employed to test the performance of FA1→3 and its convergence rate and the proposed algorithm proved to be efficient.

Four new variants of compact firefly algorithm (cFA) were proposed by (Tighzert et al. 2019). The swarm of FA is compacted and represented by a Probability of Density Function (PDF). Another two versions of compact Lévy-flight firefly algorithm (cLFA) were also introduced.

A novel FA approach proposed by (Bacanin et al. 2021) addresses issues of the basic FA by assimilating the following procedures: Explicit exploration mechanism based on the solution's exhaustiveness; gBest chaotic local search (CLS) strategy. The modified algorithm is named as chaotic FA with enhanced exploration (CFAEE).

3.2 Updating Strategy of FA

(Mohammadi et al. 2013) modified the classic FA by two strategies: 1) improving the diversity of the population through 2 mutations and 3 cross over operations. 2) Encouraging the total firefly population to move toward the best promising local or global individual. For a firefly i attracted by another firefly j , the search is updated to be in the vicinity of x_j , as given by $x_i = x_j + \beta (x_j - x_i) + \alpha \epsilon$. If only improved solutions are found after the update it will be accepted. Since the

update is done based on the location of x_j , the exploration property of points in between the two solutions will not be done, and it will be trapped in local optimum solution provided x_j is a local solution, and the step lengths are small. Through iterations, the solutions will be forced to be in a neighbourhood of the best solution. The diversity of the solutions will also be low. Adaptive Modified Firefly Algorithm (AMFA) is employed to achieve an optimal operational planning with regard to cost minimization. Performance of the proposed method is verified using a typical Micro Grid-MG.

A similar modification in the vicinity of the brighter firefly is given in (Kazemzadeh and Kazemzadeh 2011). They proposed two updating formulas, with and without division, as the authors name them. The updating formula, without division, is given by $x_i = x_j + \alpha \epsilon$. Once the fireflies are sorted according to their brightness, increasing with their index, the updating formula will be defined by $x_i = x_j + \alpha_j \epsilon$, which will decrease α as the brightness increases and which will give a good intensification or exploitation property. Different types of truss structures under stress, displacement and buckling constraints are carried out and numerical results are compared to the previously reported results in the literature

For a data clustering problem, the standard firefly algorithm is modified by (Hassanzadeh and Meybodi 2012) by using global optima in firefly movement. when a firefly compared with other firefly instead of the one firefly being allowed to influence and to attract its neighbors, global optima (a firefly that have maximum or minimum value) in each iteration can be allowed to influence others and affect in their movement. in the MFA, when a firefly compare with correspond firefly, if the correspond firefly be brighter, the compared firefly will move toward correspond firefly, considered by global optima. The authors used firefly algorithm to find optimal cluster centers and then initialized the k-means algorithm with these centers to refine the centers. This method applies to 5 datasets.

(Amaya et al. 2014) modified the standard FA so that the convergence time of the algorithm is reduced, while increasing its precision (i.e. it is able to converge with less error). They proposed three variants of FA: 1) They made α, γ to change dynamically by introducing a new equation, 2) a firefly (x_i) is automatically attracted to the vicinity of another one (x_j), by means of new a new updating equation, striving to speed up the optimization process 3) Dividing over the firefly index. This means that the fireflies attracted to the best solution found so far, would be located in a smaller vicinity region, thus favoring the intensity of the search

3.3 Hybridizing with Other Optimization Techniques

A novel firefly algorithm with deep learning named Gender-Based Deep Learning Firefly Algorithm Gen-DLFA is proposed by (Zhang et al. 2021) to generate structural test data. In this method, initially the population is divided into male group and female group. each male

firefly is attracted by another randomly selected female firefly, representing the global search ability, while female fireflies implement local search guided by the general center firefly constructed through certain times of one-dimensional deep learning. With further equations proposed, the movement of firefly is updated. Later, Chaos search is implemented around the current best solutions to generate k candidate solutions to improve accuracy and population diversity. Their simulation results show that the proposed algorithm can achieve better performance in terms of success coverage rate, coverage time, and diversity of solutions.

A two hybrid meta-heuristic algorithms based on the TS algorithm and firefly algorithm, which are called 'TSFFI' and 'TSFFII' is proposed by (Rohaninejad et al. 2015). In these hybrid algorithms, the TS algorithm is used to improve the sub-problem of capacitated job shop scheduling problem (CJSP), determining the sequence of operations, and the firefly algorithm is employed to improve the sub- problem of determining the maximum amount of machines overtime in each period.

(Panda et al. 2018) hybridized an evolutionary population-based algorithm (IWFA) invasive weed optimization technique with FA to improve the position of the current weed population in the colony. The proposed hybridization IWFO maintains the efficient balance between exploration and exploitation because of adopting spatial dispersion property of IWO algorithm and movement property of the firefly, to explore new feasible region and exploits the new population of the weed colony.

(Lieu et al. 2018) presented a novel adaptive hybrid evolutionary firefly algorithm (AHEFA) for shape and size optimization of truss structures under multiple frequency constraints. This algorithm is a hybridization of the differential evolution (DE) algorithm and the firefly algorithm (FA). An automatically adapted parameter computed from the deviation of objective function between the best individual and the whole population in the previous generation is used to select an appropriate mutation operator for the performance in the mutation phase.

Symmetric TSP (Travelling Salesmen Problem) was addressed by FA (Jati and Suyanto 2011) The proposed EDFA (Evolutionary discrete firefly algorithm) use permutation representation where city is represented by element array and index denotes tour order. Each inversion mutation of firefly use 'm' generated moves. The results were simulated on some standard TSP library problem where proposed EDFA outperform memetic algorithm.

(Hassanzadeh et al. 2012) proposed using FA for speech recognition by training SEFNN (structure equivalent fuzzy neural network) parameter. the generalization ability of fuzzy neural network is improved by firefly algorithm. the result shows that Speech recognition done by hybrid algorithm has higher recognition rate than classical fuzzy neural network trained by PSO.

A mutated hybrid firefly (MHF) algorithm is proposed by (Satapathy et al. 2016) where global optima concept adaptive randomization parameter and mutation of fireflies are modified. In this paper the improved harmonic search (IHS) technique is used for mutation with firefly technique to improve the diversity of the fireflies. MHF is tested on parametric error minimization of microgrid dynamics. The optimization technique establishes effectiveness with fast convergence as compared with conventional and without optimized system responses.

A new hybridized technique named as HAdFA (Hybrid Adaptive Firefly Algorithm) was developed by (Gayathri Devi et al. 2022) to solve flexible job shop scheduling. The paper adopted two adaptive strategies, i.e., an adaptive randomization parameter (α) and an effective heterogeneous update rule for fireflies to strike a balance between diversification and intensification. An enhanced local search algorithm, Simulated Annealing (SA), is hybridized with Adaptive FA to explore the local solution space more efficiently and the authors demonstrated that their HAdFA is superior to other existing algorithms in literature.

Table 2: Modified Variants of FA

S.No:	Authors	Parameters tuned	Proposed Algorithm	Test problems	Developing software	Applications
1.	Fister et al. 2012	α	Memetic FA (MFA)	Culberson random graph generator	Tabucol and HEA	
2.	Yu et al 2013	α	Self-Adaptive Step FA (SASFA).	Sixteen benchmark functions (continuous optimization)	Matlab	
3.	Yu et al 2014	α	Wise Strategy for Step Setting (WSSFA)	20 benchmark test functions	Matlab	

4.	Cheung et al 2014	α , Updating of fireflies, γ	Adaptive FA (AdFA)	twelve benchmark functions		
5.	Yu et al 2015	α	Variable Step Size Firefly Algorithm (VSSFA)	sixteen benchmark functions (continuous optimization)	Matlab	
6.	Liu et al 2020	α	Dynamic Adaptive Firefly Algorithm With Gaussian Distribution (GDAFA).	i) 18 classical test functions with different optimization features. ii) two engineering design constraint optimization problems	Matlab	
7.	Baykasoğlu and Ozsoydan 2015	γ	Adaptive Firefly Algorithm	Benchmark mechanical design optimization problems	Matlab	
8.	Carbas 2016	β, α	EFFA	105-member regular steel space frame and a 568-member unbraced steel space frame	Matlab	Civil
9.	Wang et al 2017	α	ApFA	Thirteen benchmark functions of continuous optimization	Matlab	Mathematics
10.	Sababha et al. 2018	α, γ	Improved FA	Unimodal and multimodal benchmark sets	Matlab	Mathematics
11.	Ghasemi et al 2022	Movement updating	FA1→3	CEC2014 test functions	-	Engineering Optimization
12.	Tighzert et al 2019	Swarm Size	cFA, cLFA	30 test functions taken from IEEE CEC2014		Robotics
13.	Mohammadi et al. 2013	Updating strategy	AMFA	Benchmark data set of MG	Matlab	Power systems Optimization
14.	Kazemzadeh and Kazemzadeh	Updating strategy	Modified FA	18 bar, 20 bar Truss structure		Structural Engineering
15.	Hassanzadeh and Meybodi 2012	Updating strategy/ hybridized FA with k-mean	Modified FA (MFA)	Iris, WDBC, Sonar, Glass and Wine data sets		Data mining/computer science

		algorithm				
16.	Amaya et al. 2014	α, γ , Updating movement, Firefly index	Modified FA	Benchmark data sets like - Jongs function		Mathematics
17.	Bacanin et al. 2021	Search space, movement	CFAEE	Congress on Evolutionary Computation (CEC) benchmark suite		
18.	Zhang et al. 2021	Hybridized with Deep learning	Gender-Based Deep Learning Firefly Algorithm Gen-DLFA	Benchmark Software test instances	Java	Software testing
19.	Lieu et al 2018	FA hybridized with DE	AHEFA	Benchmark Truss structures	Matlab	Civil
20.	Rohaninejad et al 2015	FA hybridized with TS	TSFFI and TSFFII	Benchmark instances	GAMS 23.4 software	Operations research/Industrial engineering
21.	Panda et al 2018	FA hybridized with IWA	IWFO	Simulations done in C	'C' program	Robot motion planning
22.	Jati and Suyanto 2011	FA hybridized with Evolution strategies	EDFA (Evolutionary discrete firefly algorithm)	TSP benchmark data		Operations Research
23.	Hassanzadeh et al 2012	FA integrated with Fuzzy Neural Network(FNN)	SEFNN	Speech samples		Speech Therapy
24.	Satapathy et al. 2016	FA hybridized with improved harmonic search (IHS)	Mutated Hybrid Firefly	Simulated power grid model	MATLAB	Power Engineering
25.	Gayathri Devi et al 2021	Adaptive FA hybridized with SA	HAdFA	Brandimarte instances of OR library	Matlab	Scheduling

IV. DISCUSSIONS AND CONCLUSION

Firefly algorithm is becoming an important and powerful method in comparison to several other meta-heuristic methods, such as GA, PSO, HS, SA, Ant Colony Optimization etc., because of its simplicity, versatility, superior efficiency. It has more flexibility to tread-off between exploration and exploitation. It means that the firefly algorithm is strong to search globally as well as local search in a design domain. Because of this property, the firefly algorithm can reach more near to the exact solution. Though the standard FA works well for many applications, it can still be improved. Given the complex nature of real-world optimization problems, the basic structure of FA has been modified to enhance its performance. The modification has been done in many

parts of the FA structure. The modified variants of FA have many applications especially for optimization problems. electrical and power system, signal and image processing, wireless sensor networking, clustering and classification, global function optimization, computer gaming, structural and mechanical engineering and many others.

In this paper, a detailed review of modified versions of firefly algorithm is presented. This review has summarized the most recent FA variants. The modifications are used to boost its performance for both continuous and non-continuous problems. The variations of FA variants have been discussed based on three classifications, namely, modified variants, hybridized variants and adaptive parameter variants. Most of the modifications are done on Randomness parameter α and Attractive Co-efficient β . In summary, this review has

justified that FA is a potentially powerful and useful tool for solving different optimization problems in a diverse range of applications. New modifications and improvements can enhance its performance even further. The authors hope that this chapter can inspire interested researchers and practitioners to carry out more research in this area and to solve more complex and challenging problems in practice.

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