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C*- Set in Topological Spaces

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ABSTRACT

In this paper, we introduce the notion of C*-set in topological spaces and study some of its properties.

***Keywords*--** gp-Open Set, αg-open Set, Topology

I. INTRODUCTION AND PRELIMINARIES

Topology is the mathematical study of shapes and spaces. Topology developed as a field of study out of geometry and set theory, through analysis of such concepts as space, dimension and transformation. In 1970, Levine initiated the study of so called generalized closed sets. The notion has been studied extensively in recent years by many topologists because generalized closed sets plays not only important role in generalization of closed sets but also they have suggested some new separation axioms, some of them have been found to be useful in computer science, digital topology and quantum physics.

Throughout this paper, X and Y denote topological spaces (X, τ) and (Y, σ) respectively. For a subset A of X , the closure, the interior and the complement of A are denoted by cl(A) , int(A) and A^C respectively.

Definition 1.1. A subset S of X is called

a) an A-set[8] if $S=G \cap F$ where G is open and F is regular closed in X,
b) a t-set[9] if int(S)=int(cl(S)),
c) a B-set [9]if $S= G \cap F$ where G is open and F is a t-set in X,
d) an α*-set [4] if int(S)=int(cl(int(S))),
e) a C-set (due to Sundaram)[7] if $S= G \cap F$ where G is g-open and F is a t-set in X,
f) a C-set (due to Hatir,Noiri, and Yuksel)[4] if $S= G \cap F$ where G is open and F is an α*-set in X.

II. C*-SET

Definition 2.1. A subset S of X is said to be a C*-set in X if $S=A \cap B$ where A is g-open and B is an $\alpha*$-set in X.

Proposition 2.2. Every g-open set in X is a C*-set in X.

Proof . Let A be g-open set in X. X is an α*-set in X. Now, $A = A \cap X$. Therefore A is a C* set in X.
However , the converse need not be true as seen from the following example.

Example 2.3 : Let X = {a,b,c} and $\tau = \{\phi, \{a\}, X\}$. Then {b, c} is a C*-set in X but not g-open in X.

Preposition 2.4 : Every $\alpha*$ –set in X is a C*-set in X .

Proof . Let B be an $\alpha*$-set in X. X is g-open in X. Now, $B = X \cap B$.
Therefore B is a C*-set in X.
However , the converse need not be true as seen from the following example.

Example 2.5 : Let (X , τ) be as in example 2.3.
The set {a} is a C*-set in X but not an α*-set in X.

Preposition 2.6 : Let A and B be C*-sets in X. Then $A \cap B$ is a C*-set in X.

Proof . Since the intersection of two g-open sets in X is g-open on X [5] and since the intersection of two α*-sets in X is an $\alpha*$- set in X [4], the proof follows.

Remark 2.7 . a) The union of two C*-sets in X need not be a C*-set in X and
(b) the complement of a C*-set in X need not be a C*-set in X
as seen from the following example.

Example 2.8 : Let X = {a, b, c } and $\tau= \{\phi, \{a\}, \{a, b\}, X\}$. In (X, τ) , the sets {a}, {b} and {c} are C*-sets but {a, c} is not a C*-set.

Proposition 2.9 (4) Every t-set in X is an α-set in X.
However, the converse need not be true.

Proposition 2.10 : Every C(S) -set in X is a C*-set in X.

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Proof : Let S be a C(S) -set in X. Then S = A $\cap$B Where A is g-open and B is a t-set in X. However, the converse need not be true as seen from the following example.

Example 2.11 : Let X = { a, b, c } and τ= {ϕ, {a,b}, X}. The set {b,c} is an α-set in X. So C*-set in X. But {b, c} is neither a t-set nor a C(S)-set in X.

Proposition 2.12 : Every C-set in X is a C*-set in X.

Proof : Let S be a C-set set in X. Then S = A $\cap$B Where A is open and B is an α*-set in X. Since every open set in g-open, we see that S is a C*-set in X.
However, the converse need not be true as seen from the following example.

Example 2.13 : Let X = { a, b, c } and τ= {ϕ, {a}, X}. The set {a,b} is a C*-set. But not a C-set in X.

Proposition 2.14 (4) : Every B-set in X is a C-set in X. However, the converse need not be true.

Proposition 2.15 (6) : Every B-set in X is a C(S)-set in X. However, the converse need not be true.

Proposition 2.16: Every B-set in X is a C*-set in X.

Proof : Follows from proposition 2.15 and proposition 2.10.
However, the converse need not be true as seen from the following example.

Example 2.17 : Let X = { a, b, c } and τ= {ϕ, {a,b}, X}. The set {a} is a C*-set but not a B-set in X.

Remark 2.18 . The notion of C-set is independent of the notion of C(S) -set as seen from the following examples.

Example 2.19 : Let X = { a, b, c } and τ= {ϕ, {a }, X}. The set {a,b} is a C(S) -set but not a C-set in X.

Example 2.20 : Let X = { a, b, c } and τ= {ϕ, {a,b }, X}. The set {b,c} is a C-set but not a C(S)-set in X.

Remark 2.21 . The notion of C(S) is independent of the notions of gp-open set and αg-open set and the notion of C*-set is independent of the notions of gp-open set and αg-open set as seen from the following example.

Example 2.22 : Let X = { a, b, c } and τ= {ϕ, {a}, X}. Then the set {b,c} is a C(S)-set and a C*-set but is neither a gp -open set nor a αg-open set in X.

Let Y = { a, b, c } and σ = {ϕ, {a}, {a,b }, X}. Then the set {a,c} is both αg-open and gp-open in (Y, σ) but is neither a C(S)-set nor a C*-set in (Y,σ).

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