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C*-Continuity in Topological Spaces

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ABSTRACT

In this paper , we introduce and study the notion C* - continuity in topological spaces.

***Keywords*--** C-Continuity, C(S)-Continuity, Topology

I. INTRODUCTION AND PRELIMINARIES

Topology is the mathematical study of shapes and spaces. Topology developed as a field of study out of geometry and set theory, through analysis of such concepts as space, dimension and transformation. In 1970, Levine initiated the study of so called generalized closed sets. The notion has been studied extensively in recent years by many topologists because generalized closed sets plays not only important role in generalization of closed sets but also they have suggested some new separation axioms, some of them have been found to be useful in computer science, digital topology and quantum physics.

Throughout this paper, X and Y denote topological spaces (X, τ) and (Y, σ) respectively. For a subset A of X , the closure, the interior and the complement of A are denoted by cl(A) , int(A) and A^C respectively.

Definition 1.1. A subset S of X is called

a) an A-set if S=G $\cap$ F where G is open and F is regular closed in X,
b) a t-set if int(S)=int(cl(S)),
c) a B-set if S= G $\cap$ F where G is open and F is a t-set in X,
d) an α*-set if int(S)=int(cl(int(S))),
e) a C-set (due to Sundaram) if S= G $\cap$ F where G is g-open and F is a t-set in X,
f) a C-set (due to Hatir,Noiri, and Yuksel) if S= G $\cap$ F where G is open and F is an α*-set in X.

Proposition 1.2 : Every C(S) -set in X is a C*-set in X.

Proof : Let S be a C(S) -set in X. Then S = A $\cap$B Where A is g-open and B is a t-set in X.

Proposition 1.3 : Every C-set in X is a C*-set in X.

Proof : Let S be a C-set set in X. Then S = A $\cap$B Where A is open and B is an α*-set in X. Since every open set in g-open, we see that S is a C*-set in X.

Proposition 1.4: Every B-set in X is a C*-set in X.

Proposition 1.5. Every g-open set in X is a C*-set in X.

II. C*-CONTINUITY

Definition 2.1. : A Map f : X $\rightarrow$Y is said to be C*-continuous if f^{-1}(G) is a C*-set in X for every open set G in Y.

Preposition 2.2 : If f : X $\rightarrow$Y is C-continuous, then f is C*-continuous.

Proof : Let G be open set in Y. If f : X $\rightarrow$Y is C-continuous, then f^{-1}(G) is a C-set in X. By proposition 1.3, f^{-1}(G) is a C*-set in X. Therefore f is C*-continuous .

However, the converse need not be true as seen from the following example.

Example 2.3. Let X = { a, b, c } , τ= {ϕ, {a}, X}, Y = {x,y} and σ ={ϕ, {x}, Y} Define f :(X, τ) $\rightarrow$ (Y, σ) by f(a) = f(b) = x and f(c) = y. Then f is C*-continuous and g-continuous but not C-continuous .

Preposition 2.4 : If f : X $\rightarrow$Y is C(S)-continuous, then it is C*-continuous

Proof : Assume that f : X $\rightarrow$Y is C(S) - continuous and let G be an open set in Y. Then f^{-1}(G) is a C(S)-set in X. By proposition 1.2 , f^{-1}(G) is a C*-set in X.
However, the converse need not be true as seen from the following example.

Example 2.5. . Let X = { a, b, c } , Y = {x,y}, τ = {ϕ, {a,b}, X} and σ ={ϕ, {x}, Y} Define f : (X, τ) $\rightarrow$ (Y, σ) by f(a) = y and f(b) = f(c) = x. Then f is C*-continuous but not C(S)-continuous.

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Proposition 2.6 (4) A B-continuous map is C(S)-continuous.
However, the converse need not be true.

Proposition 2.7 . A B-continuous map is C*-continuous.

Proof : Let f : X $\rightarrow$Y be B - continuous and let G be an open set in Y. Then f^{-1}(G) is a B-set in X. By proposition 1.4 , f^{-1}(G) is a C*-set in X. Therefore f is C*-continuous.

However, the converse need not be true as seen from the following example.

Example 2.8. Let X = { a, b, c } , τ= {ϕ, {a}, X}, Y = {x,y} and σ ={ϕ, {x}, Y} Define f :(X, τ) $\rightarrow$ (Y, σ) by f(a) = f(b) = x and f(c) = y. Then f is C* -continuous .
But not B-continuous.

Proposition 2.9 [3] . A B-continuous map is C-continuous.
However, the converse need not be true.

Theorem 2.10 [4].A map f : X $\rightarrow$Y is g-continuous if and only if f^{-1}(G) is g-open in X for each open set G in Y.

Proposition 2.11 . A g-continuous map is C(S)-continuous.

Proof : Let f : X $\rightarrow$Y be g-continuous ant let G be an open set in Y. Then S = f^{-1}(G) is g-open in X . Now S = S $\cap$X where S in g-open and X, the whole space, which is clearly a t-set in X. Therefore S is a C(S)-set in X. Hence f is C(S)-continuous.

However, the converse need not be true as seen from the following example.

Example 2.12. Let X = { a, b, c } , Y = {x,y}, τ = {ϕ, {a} X} and σ ={ϕ, {x}, Y} Define f : (X, τ) $\rightarrow$ (Y, σ) by f(a) = y and f(b) = f(c) = x. Then f is C(S) - continuous and B-continuous but not g-continuous.

Proposition 2.13 . A g-continuous map is C*-continuous.

Proof : Let f : X $\rightarrow$Y be g- continuous ant let G be an open set in Y. Then f^{-1}(G) is g-open in X . By proposition 1.5, f^{-1}(G) is a C*-set in X. Therefore f is C*-continuous.
However, the converse need not be true as seen from the following example.

Example 2.14 : Let f : X $\rightarrow$Y be as in example 2.12. Then f is C*-continuous but not
g-continuous.

Remark 2.15 : The notions of C-continuity and C(S)-continuity are independent as seen from the following examples.

Example 2.16. Let X = { a, b, c } and τ = {ϕ, {a} X}. Let Y = X be with discrete topology. Let f : X $\rightarrow$Y be the identity map. Then f^{-1}({a,c}) is not a C-set but the collection of all C(S) – sets in X is the power set P(X). Therefore f is C(S)-continuous but not C-continuous.

Example 2.17. Let X = { a, b, c } and τ = {ϕ, {a,b} X}. Let Y = X be with discrete topology. Let f : X $\rightarrow$Y be the identity map. Then f^{-1}({a,c}) is not a C(S)-set but the collection of all C-sets in X is P(X) .
Therefore f is C-continuous but not C(S)-continuous.

Remark 2.18 . The notion of gp-continuity is independent of C(S)-continuity and C*- continuity as seen from the following examples.

Example 2.19. Let X = { a, b, c } and Y = {x,y}, τ = {ϕ, {a}, X} and σ = {ϕ, {x}, Y}. Define f :(X, τ) $\rightarrow$ (Y, σ) by f(a) = y and f(b) = f(c) = x. Then f is C(S)-continuous and C*-continuous but is neither gp-continuous nor αg-continuous.

Example 2.20. Let X = { a, b, c } , Y = {x,y}, τ = {ϕ, {a}, {a,b}, X} and σ = {ϕ, {x}, Y}. Define f : (X, τ) $\rightarrow$ (Y, σ) by f(a) = f(c) = x and f(b) = y.
Then f is C(S)-continuous and C*-continuous but is neither gp-continuous nor
αg- continuous. Then f is gp-continuous and αg-continuous but is neither
C*-continuous nor C(S)-continuous.

Remark 2.21 : From Devi [2], Arockiarani [1] and from results obtained in this section, we have the following diagram.

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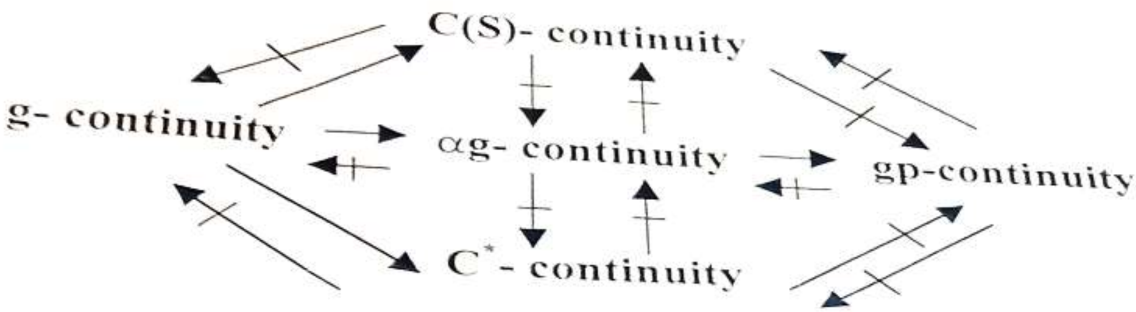
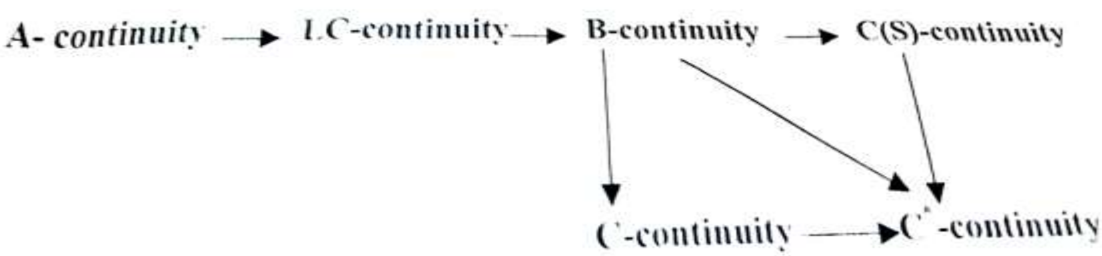


Remark 2.22 : From Tong [5], Hatir, Noiri and Yuksel [3] and from the results obtained in this section, we have the following implications.



However, none of the implications is reversible and that C-continuity and C(S)-continuity are independent.

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