Thermal Stress Analysis of Composite Laminates using Trigonometric Shear Deformation Theory and Finite Element Method

S M Shiyekar[1] and Anil P Patil[2]
[1]Professor, Department of Civil Engineering, D Y Patil College of Engineering, Akurdi, Pune, INDIA
[2]Former PG Scholar, Department of Civil Engineering, Rajarambapu Institute of Technology, Sangli, INDIA

[2]Corresponding Author: patilanil4489@gmail.com

Received: 10-09-2023 Revised: 28-09-2023 Accepted: 16-10-2023

ABSTRACT

Laminated composite materials offer a versatile design approach for achieving the desired levels of stiffness and strength by selecting specific lamination schemes. The Trigonometric Shear Deformation Theory (TrSDT) effectively addresses the appropriate distribution of transverse shear strains throughout the plate thickness while maintaining stress-free boundary conditions on the plate's top surfaces. Consequently, there is no need for a shear correction factor.

In this research paper, we use the Trigonometric Shear Deformation Theory (TrSDT) that takes into account the influence of transverse shear deformation. The in-plane displacement field incorporates a sinusoidal function with respect to the thickness coordinate to accommodate the effects of shear deformation. Theories that involve trigonometric functions based on the thickness coordinate in the displacement fields are collectively referred to as Trigonometric Shear Deformation Theories (TrSDTs).

In the present study, we conduct a thermal stress analysis of Laminated Composite Plates using the TrSDT. This theory eliminates the need for shear correction factors and provides a more accurate distribution of interlaminar stresses compared to other methods like CPT and FOST. We assess deflection and stress at various locations and for different aspect ratios under thermal loads using TrSDT. Stress evaluations are carried out analytically, and the results are validated by comparing them with existing findings from the literature.

To further verify our findings, we model a composite laminate under thermal loads using the commercial Finite Element Method tool ABAQUS, and our results are validated against those obtained with the TrSDT for plates with simply supported boundary conditions.

Keywords-- Composite Laminated Plates, Trigonometric Shear Deformation Theory (TrSDT), Thermal Loading, ABAQUS

I. INTRODUCTION

Plate structures play a pivotal role as primary load-bearing elements in the realm of Structural Mechanics, finding widespread applications in terrestrial, naval, and aeronautical engineering. These plates often endure substantial in-plane compression forces and/or shear loads. What makes them particularly versatile is their mechanical properties in various directions and their exceptional strength-to-weight ratios, which allow for tailored configurations. Furthermore, they exhibit a range of unique characteristics, encompassing resistance to corrosion, high damping capacity, temperature resilience, and a low coefficient of thermal expansion.

These exceptional attributes have led to an expanded utilization of advanced composite materials in structures exposed to thermal environments. The use of highly fiber-reinforced materials empowers designers to finely control the structure's stiffness and strength. Composite laminates are formed by stacking layers of different composite materials and/or adjusting fiber orientations, with their planar dimensions being significantly larger than their thickness. In many cases, laminates are employed in applications that demand both axial and bending strength, treating them effectively as plates.

Numerous plate theories are accessible to describe the static and dynamic behavior of such plates. The choice of a specific plate theory depends on factors like plate geometry and material properties, with one theory being more suitable than another in various scenarios. Understanding the distinctions between these theories and their practical application is of great significance to engineers involved in plate structures and researchers engaged in advancing our understanding of plate behavior.

Shimpi and Ghugal [1] introduced a novel layerwise trigonometric shear deformation theory for the analysis of two-layered cross-ply laminated beams. This theory not only streamlines the number of primary variables, reducing them to even fewer than those in the first-order shear deformation theory, but also eliminates the need for a shear correction factor. Ghugal and Shimpi [2] provided an overview of displacement and stress based refined theories for isotropic and anisotropic laminated plates. They discussed various equivalent single-layer and layerwise theories for laminated plates, highlighting their advantages and disadvantages. The paper also referenced exact elasticity solutions for plate problems where available, addressing various critical issues related to plate theories based on the literature review. Matsunaga [3] described a two-dimensional global higher-order deformation theory for evaluating inter-laminar stresses and displacements in cross-ply multilayered composite and sandwich plates subjected to thermal loads. Chen et al. [4] discussed a new higher-order shear deformation theory based on a global-local superposition technique, emphasizing the satisfaction of free surface conditions, geometric continuity conditions at interfaces, and stress continuity conditions at interfaces. Kapuria and Achary [5] introduced a new efficient higher-order zigzag theory for laminated plates subjected to thermal loading. This theory modifies the third-order zigzag model by incorporating a layerwise variable approximation for deflection, explicitly considering the transverse thermal strain. Ferreira et al. [6] discussed a trigonometric shear deformation theory for symmetric composite plates, discretized using a meshless method based on global multiquadric radial basis functions. Shimpi [7] explored the First-Order Shear Deformation Theory, particularly those proposed by Reissner and Mindlin, which remain in use due to their simplicity. Two new displacement-based first-order shear deformation theories were introduced, each involving only two unknown functions, in contrast to the three functions used in Reissner's and Mindlin's theories. Zhang et al. [8] presented a review of the recent developments in finite element analysis for laminated composite plates from 1990 onwards. The review covered finite elements based on various laminated plate theories for free vibration, static analysis, buckling, postbuckling analysis, geometric nonlinearity, large deformation analysis, and failure and damage analysis of composite laminated plates. Roylance [9] outlined the mechanics of fiber-reinforced laminated plates, elucidating a computational approach that connects in-plane strain and curvature with tractions and bending moments imposed on the laminate. The paper reviewed constitutive relations for isotropic materials and demonstrated the straightforward extension to transversely isotropic composite laminae. Kant and Shiyekar [10] presented a comprehensive analytical model that accounted for shear deformation and transverse normal thermal strains in the thermal stress analysis of cross-ply laminates subjected to linear or gradient thermal profiles across the laminate's thickness. The model used twelve degrees of freedom to expand the primary displacement field in the thickness direction. The resulting equilibrium equations, based on higher-order shear and normal deformation theory, were variationally consistent and derived using the principle of virtual work. Numerical results for displacements and stresses were compared with classical plate theory, first-order shear deformation theory, and higher-order shear deformation theory. Mantari et al. [11] developed a trigonometric shear deformation theory for isotropic and composite laminated and sandwich plates, introducing a parameter 'm' to align results with three-dimensional elasticity bending solutions. This theory effectively distributes transverse shear strains across the plate thickness, eliminating the need for a shear correction factor. Tornabene et al. [12] employed the Generalized Differential Quadrature (GDQ) method to study laminated composite degenerate shell panels, including rectangular and annular plates. The method enabled the determination of stress profiles through the thickness of plates without specifying specific equations for these plate types, making the theoretical treatment general. Thai et al. [13] introduced an inverse tangent shear deformation theory (ITSDT) for the dynamic, free vibration, and buckling analysis of laminated composite and sandwich plates. Ghugal and Kulkarni [14] addressed thermal stresses and displacements in orthotropic, two-layer antisymmetric, and three-layer symmetric square cross-ply laminated plates subjected to nonlinear thermal loads through the thickness of the plates. They employed trigonometric shear deformation theory.

Srinivasan and Rao [15] presented Bending, vibration and buckling of simply supported thick orthotropic rectangular plates and laminates. Pagano [16] developed exact Solutions for Rectangular Bidirectional Composites and Sandwich Plates. Karama [17] presented A new theory for laminated composite plates. Reddy [18] developed third order shear deformation theory. Kulkarni and Kapuria [19] developed A new discrete Kirchhoff quadrilateral element based on the third-order theory for composite plates. Shaikh and Chakrabarti [20] presented a New Plate Bending Element Based on Higher Order Shear Deformation Theory for the Analysis of Composite Plates. Chen and Wu [21] presented a new higher-order shear deformation theory and refined beam element of composite laminates. Sahooand and Singh [23] developed a new shear deformation theory for the static analysis of laminated composite and sandwich plates. Kant and Swaminathan [24] developed analytical solutions for the static analysis of laminated composite and sandwich plates based on a higher order refined theory. Reddy and Liu [25] presented a higher-order shear deformation theory of laminated elastic shells.

In this paper, to confirm our results, we conduct a thermal analysis of a composite laminate in ABAQUS [22], a commercial Finite Element Method software. Subsequently, we compare our findings with those obtained through the application of the Trigonometric Shear Deformation Theory (TrSDT) to plates with boundary conditions characterized as simply supported.

II. FORMULATION OF TRIGONOMETRIC SHEAR DEFORMATION THEORY (TrSDT)

We use a Trigonometric Shear Deformation Theory (TrSDT) that not only considers the impact of transverse shear deformation but also accounts for the influence of transverse normal strain. In this theory, the in-plane displacement field is defined using a sinusoidal function with respect to the thickness coordinate to accommodate the effects of shear deformation. Additionally, a cosine function in the thickness coordinate is employed in the transverse displacement to address the impact of transverse normal strain. The governing equations and boundary conditions for this theory are derived through the application of the principle of virtual work.

Consider a rectangular cross-ply laminated plate with dimensions length *a*, width *b*, and total thickness *h*, consisting of orthotropic layers as depicted in Fig. 1. Each layer's material is assumed to possess a plane of material property parallel to the *x-y* plane. The coordinate system is aligned so that the plate's mid-plane coincides with the x-*y* plane, while the *z*-axis is perpendicular to the middle plane. The upper surface of the plate, situated at *z* = -*h* / 2, experiences a constant thermal load denoted as *T* (*x*, *y*, *z*). The plate's region is defined within the right-handed Cartesian coordinate system of (*x*, *y*, *z*).

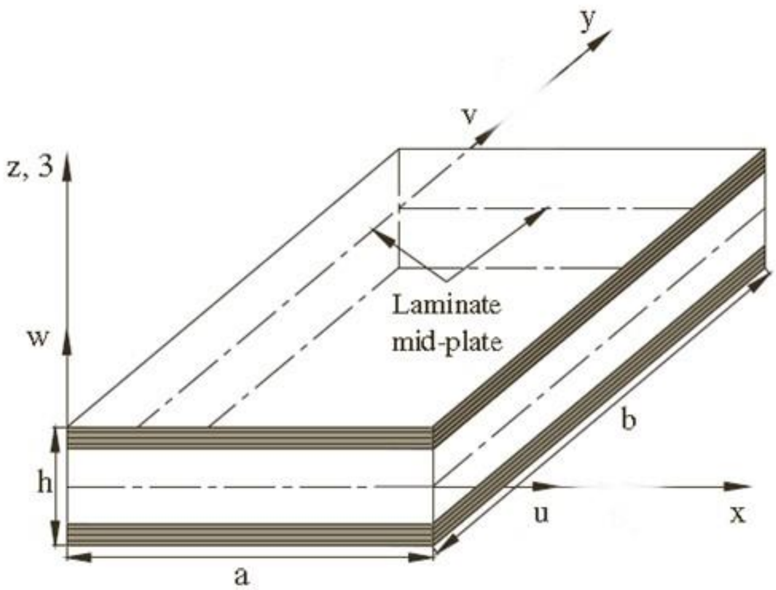

Figure 1: Laminae with reference axes

2.1 The Displacement Field

In accordance with plate theory, the displacement field for a plate with a global thickness denoted as *h* can be defined as follows:

$$u(x,y,z)=u_0(x,y)-z\frac{\partial wo(x,y)}{\partial x}+\sin\frac{\pi z}{h}\emptyset x(x,y)$$
$$v(x,y,z)=v_0(x,y)-z\frac{\partial wo(x,y)}{\partial y}+\sin\frac{\pi z}{h}\emptyset x(x,y) \qquad (1)$$
$$w(x,y,z)=w_0(x,y)$$

where, *u* and *v* are the in-plane displacements at any point (*x*, *y*, *z*). u_0 and v_0 signify the in-plane displacement of the point (*x*, *y*, *0*) on the mid-plane, *w* is the transverse deflection, f_x and f_y are the rotations of the normal to the mid-plane about the *y* and *x* axes respectively. The strain-displacement relationships is given as,

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix}=\begin{Bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\\ \frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\\ \frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\end{Bmatrix} \qquad (2)$$

Strains are expressed for symmetric laminates as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \sin\frac{\pi z}{h} \begin{Bmatrix} {\varepsilon_{xx}}^{(s)} \\ {\varepsilon_{yy}}^{(s)} \\ {\gamma_{xy}}^{(s)} \end{Bmatrix} + z \begin{Bmatrix} {\varepsilon_{xx}}^{(z)} \\ {\varepsilon_{yy}}^{(z)} \\ {\gamma_{xy}}^{(z)} \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \frac{\pi}{h}\cos\frac{\pi z}{h} \begin{Bmatrix} \emptyset_y \\ \emptyset_x \end{Bmatrix} \tag{3}$$

where,

$$\begin{Bmatrix} {\varepsilon_{xx}}^{(s)} \\ {\varepsilon_{yy}}^{(s)} \\ {\gamma_{xy}}^{(s)} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \emptyset_x}{\partial x} \\ \frac{\partial \emptyset_y}{x \partial y} \\ \frac{\partial \emptyset_x}{\partial y} + \frac{\partial \emptyset_y}{\partial x} \end{Bmatrix}; \qquad \begin{Bmatrix} {\varepsilon_{xx}}^{(z)} \\ {\varepsilon_{yy}}^{(z)} \\ {\gamma_{xy}}^{(z)} \end{Bmatrix} = \mathrm{z} \begin{Bmatrix} -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial y^2} \\ -2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix} \tag{4}$$

Neglecting s_z for each layer, the stress-strain relations in the orthotropic local coordinate system can be expressed as

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \\ \tau_{23} \\ \tau_{31} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{33} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & C_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{31} \end{Bmatrix} \tag{5}$$

This equation is referred to as the Constitutive Relationship for the material. In this equation, subscripts 1 and 2 represent the fiber and the direction normal to the fiber directions, 3 denotes the direction normal to the plate. Through appropriate coordinate transformations, the stress-strain relationships for the k^{th} layer in the global *x-y-z* coordinate system can be derived as follows:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} & 0 & 0 \\ Q_{12} & Q_{22} & Q_{26} & 0 & 0 \\ Q_{16} & Q_{26} & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & Q_{45} \\ 0 & 0 & 0 & Q_{45} & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \tag{6}$$

2.2 Governing Equations

The governing equations are derived from the principle of virtual displacements.

$$\frac{\partial \emptyset_x}{\partial x} + \frac{\partial \emptyset_y}{\partial y} + q(x,y) = 0$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - \emptyset_x = 0$$

$$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - \emptyset_y$$

$$\frac{\partial Ns_{xx}}{\partial x} + \frac{\partial Ns_{xy}}{\partial y} - \frac{\pi}{h} Tc_{xz} = 0$$

$$\frac{\partial Ns_{xy}}{\partial y} + \frac{\partial Ns_{xy}}{\partial x} - \frac{\pi}{h} Tc_{yz} = 0$$

(7)

Substituting stress resultants in terms of displacements in the governing equilibrium equations, we can obtain the displacements, strains and stresses induced.

III. RESULTS AND DISCUSSION

In this section, examples of composite laminated plate are presented to show accuracy and applicability of ABAQUS under static loading. The results obtained are compared with published results.

Example 1: Numerical investigation of square composite laminated plates under mechanical loading (b=a)

A simply supported square laminated composite plate having dimension 1mx1m of side 'a' and 'b' thickness 'h' for various aspect ratios is composed of three equally layers oriented at $[0^0/90^0/0^0]$ subjected to doubly sinusoidal load 10 kN/m^2 as amplitude for m = 1, n = 1. The result of displacement and stress using ABAQUS software and with the literature are verified. The material properties are $(\boldsymbol{\upsilon}) = 0.25, \frac{E_1}{E_2} = 25\ ;\ G_{12} = G_{13} = 0.5E_2;\ G_{23} = 0.2E_2$

Formulae used for dimensionless maximum transverse deflection and stress are as follows.

$$\bar{w} = w\left(\frac{a}{2}, \frac{b}{2}, 0\right)\left(\frac{Eh^3}{a^4 q_0}\right) * 10^2$$

$$\bar{\sigma}_{xx} = \sigma_{xx}\left(\frac{a}{2}, \frac{b}{2}, \frac{h}{2}\right)\left(\frac{h^2}{a^2 q_0}\right)$$

$$\bar{\sigma}_{yy} = \sigma_{yy}\left(\frac{a}{2}, \frac{b}{2}, \frac{h}{6}\right)\left(\frac{h^2}{a^2 q_0}\right)$$

$$\bar{\sigma}_{xy} = \sigma_{xy}\left(0, 0, \frac{h}{2}\right)\left(\frac{h^2}{a^2 q_0}\right)$$

The results and the contour plots for transverse deflection, normal stresses and in-plane shear stress are shown in below Table 1 and Fig. 2.

Table 1: Non-dimensional transverse deflection, normal stresses and in-plane shear stress of simply supported symmetric square composite laminated plate [$0^0/90^0/0^0$] under SSL for a/h=100 (b=a)

a/h	Theory	$\overline{w}$ $(\frac{a}{2},\frac{b}{2},0)$	$\overline{\sigma_{xx}}$ $(\frac{a}{2},\frac{b}{2},\frac{h}{2})$	$\overline{\sigma_{yy}}$ $(\frac{a}{2},\frac{b}{2},\frac{h}{6})$	$\overline{\tau_{xy}}$ $(0,0,\frac{h}{2})$
100	Present TrSDT	0.434054 [-0.15]	0.539014 [0.002]	0.180521 [-0.26]	-0.0213483 [-0.25]
	Present ABAQUS [22]	0.4254 [-2.14]	0.5257 [-2.47]	0.0258 [-85.74]	0.02000 [-6.10]
	3D Exact [16]	0.4347	0.539	0.181	0.0213
	Sahooand and Singh [23]	0.4343 [-0.09]	0.5448 [1.07]	0.182 [0.55]	0.0215 [0.93]
	Mantari [11]	0.4353 [0.14]	0.539 [0]	0.181 [0]	0.0214 [0.46]
	Karama [17]	0.435 [0.067]	0.538 [-0.187]	0.18 [-0.55]	0.0213 [0]
	Reddy [18]	0.4342 [-0.11]	0.539 [0]	-	-
	Kulkarni and Kapuria [19]	0.4349 [0.05]	0.5403 [0.24]	0.181 [0]	0.0214 [0.46]
	Sheikh and Chakrabarti [20]	0.4350 [0.07]	0.5496 [1.9]	0.1828 [0.99]	0.0215 [0.93]
	Kant and Swaminathan [24]	0.4343 [-0.09]	0.5392 [0.04]	0.1807 [-0.16]	0.0214 [0.46]

[ ] % error w.r.t. 3D Exact

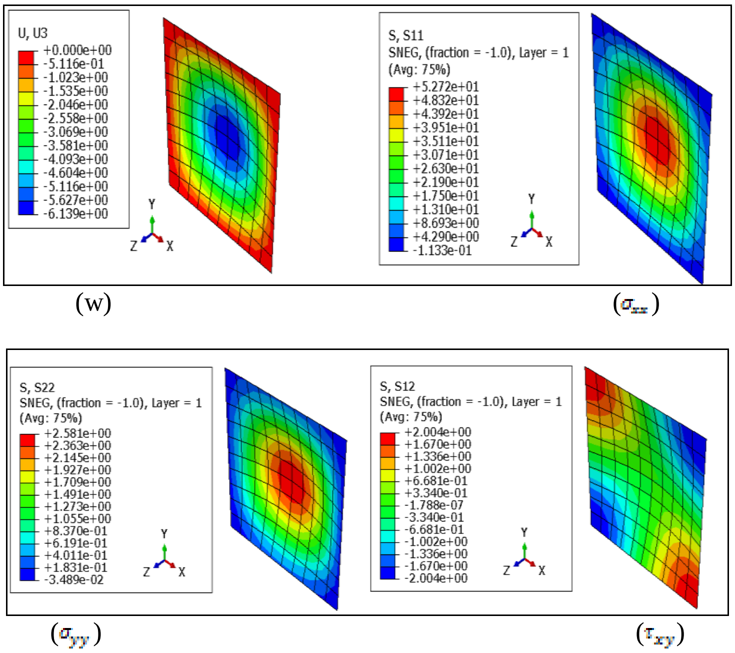

Figure 2: Contour plots for dimensional transverse deflection, normal stress and in-plane shear stress for simply supported symmetric square composite laminated plate [$0^0/90^0/0^0$] under SSL for a/h=100 (b=a) in ABAQUS

Example 2: Numerical investigation of rectangular composite laminated plates under mechanical loading (b=3a)

A simply supported rectangular composite laminated plate having dimension 1mx1m of side 'a' and 'b' thickness 'h' for various aspect ratios is composed of three equally layers oriented at [$0^0/90^0/0^0$] subjected to doubly sinusoidal load 10 kN/m^2 as amplitude for m=1, n=1.The result of displacement and stress using ABAQUS software and with the literature are verified. The material properties are

$$(\boldsymbol{v}) = 0.25, \frac{E_1}{E_2} = 25\ ;\ G_{12} = G_{13} = 0.5E_2;\ G_{23} = 0.2E_2$$

Formulae used for non-dimensionalsed maximum transverse deflection and stress are as per previous problem. The results as shown following Table 2.

Table 2: Non-dimensionalized transverse deflection, normal stresses and in-plane shear stress in three-layer [$0^0/90^0/0^0$] rectangular composite laminated plate under sinusoidal load (b = 3a) for a/h=100

a/h	Theory	$\bar{w}$ $(\frac{a}{2},\frac{b}{2},0)$	$\overline{\sigma_{xx}}$ $(\frac{a}{2},\frac{b}{2},\frac{h}{2})$	$\overline{\sigma_{yy}}$ $(\frac{a}{2},\frac{b}{2},\frac{h}{6})$	$\overline{\tau_{xy}}$ $(0,0,\frac{h}{2})$
100	Present TrSDT	0.506824 [-0.23]	0.623975 [0.0]	0.0252759 [0.0]	0.0083118 [0.4]
	Present ABAQUS [22]	0.4973 [-2.10]	-	-	-
	3D Elasticity [15]	0.5080	0.624	0.025	0.0083
	Mantari [11]	0.5081 [0.0]	0.624 [0.0]	0.025 [0.0]	0.0083 [0.4]
	Reddy and Liu [25]	0.5070 [0.2]	0.624 [0.0]	0.025 [0.0]	0.0083 [0.4]

[ ] % error w.r.t. 3D Elasticity

Example 3: Numerical investigation of composite laminated plates under thermal loading

A simply supported laminated composite plate having dimension 1mx1m of side 'a' and thickness 'h' is composed of two and three equally layers oriented at [$0^0/90^0$] and [$0^0/90^0/0^0$] respectively subjected to doubly sinusoidal temperature load of 30^0c as amplitude for m=1, n=1.The result of displacement and stress are derived using ABAQUS software and with the literature are verified. Following are the material properties,

$$\frac{E_1}{E_2} = 15G_{12}\ ;\ G_{13} = 0.5E_T\ ;\ G_{23} = 0.2E_T\ ;\ \frac{\alpha_1}{\alpha_0} = 0.015\ ;\ \frac{\alpha_2}{\alpha_0} = 1.0$$

$$v_{12} = 0.3 v_{22} = 0.49$$

Results are presented in Tables 3 and 4.

Table 3 In-plane displacement and normal stresses in Composite Laminated Square Plate for all edges simply supported composite laminated square plate [$0^0/90^0/0^0$] under sinusoidal temperature loading for a/h = 10

a/h	Qty	Source	Result
10	*u*	Present ABAQUS [22]	- 0.1635 [0.3]
		Exact [16]	-0.163
		Present TrSDT	- 0.170801
	v	Present ABAQUS [22]	-0.3916 [2.15]
		Exact [16]	0.4
		Present TrSDT	0.414125
	s_x	Present ABAQUS [22]	3.36
		Exact [16]	3.4
	s_y	Present ABAQUS [22]	1.51
		Exact [16]	1.39

[ ] % error w.r.t. Exact

Table 4 In-plane displacement and normal stresses in Composite Laminated Square Plate for all edges simply supported composite laminated square plate [$0^0/90^0$] under sinusoidal temperature loading for a/h = 10

a/h	Qty	Source	Result
10	*u*	Present ABAQUS [22]	- 0.313 [15.24]
		Exact [16]	-0.2716
	v	Present ABAQUS [22]	-0.237 [-12.22]
		Exact [16]	0.27

[ ] % error w.r.t. Exact

IV. CONCLUSIONS

The conclusions drawn from the analysis in the present study are as follows:

1. When analyzing square and rectangular composite laminated plates with the [$0^0/90^0/0^0$] orientation under mechanical doubly sinusoidal loading, we calculated non-dimensional transverse displacement, normal stresses, and shear stress at their maximum absolute values. The results obtained using ABAQUS and TrSDT for non-dimensional transverse displacement were found to closely match with analytical results, for various aspect ratios under simply supported loading. ABAQUS provided better results for both displacement and stresses.
2. When analyzing square and rectangular composite laminated plates with the [$0^0/90^0$] and [$0^0/90^0/0^0$] orientations under thermal doubly sinusoidal loading, we calculated non-dimensional transverse displacement, normal stresses, and shear stress at their maximum absolute values. Like the mechanical loading analysis, the results from ABAQUS and TrSDT for non-dimensional transverse displacement were in good agreement with Exact results for various aspect ratios under simply supported loading. Once again, ABAQUS produced more accurate results for displacement and stresses.
3. In all the analyzed problems of composite laminated plates, ABAQUS provided increasingly accurate

results for displacements and stresses as the mesh size decreased.

4. In the case of composite laminated plates, the numerical values of displacements increased as the aspect ratios increased, indicating that thinner plates exhibited greater displacement.

REFERENCES

[1] Shimpi R.P. & Ghugal Y.M. (2001). A new layerwise trigonometric shear deformation theory for two-layered cross-ply beams. *Journals of Composites Science and Technology, 61*, 1271–1283.

[2] Ghugal Y. M. & Shimpi R.P. (2002). A review of refined shear deformation theories of isotropic and anisotropic laminated plates. *Journal of Reinforced Plastics and Composites*, *21*, 775-815.

[3] Hiroyuki Matsunaga. (2004). A comparison between 2-D single-layer and 3-D layerwise theories for computing interlaminar stresses of laminated composite and sandwich plates subjected to thermal loadings. *Composite Structures*, *64*, 161–177.

[4] Chen W. & Wu Z. (2005). A new higher-order shear deformation theory and refined beam element of composite laminates. *Acta Mech Sinica*, *21*, 65–69.

[5] Kapuria S. & Achary G.G.S. (2004). An efficient higher order zigzag theory for laminated plates subjected to thermal loading. *International Journal of Solids and Structures, 41*, 4661–4684.

[6] Ferreira A.J.M., Roque C.M.C. & Jorge R.M.N. (2005). Analysis of composite plates by trigonometric shear deformation theory and multi quadrics. *Computers and Structures, 83*, 2225–2237.

[7] Shimpi R.P. (2007). New first-order shear deformation plate theories. *Journal of Applied Mechanics*, *74*, 523-533.

[8] Zhang Y.X. & Yang C.H. (2009). Recent developments in finite element analysis for laminated composite plates. *Composite Structures, 88*, 147–157.

[9] Roylance David. (2010). Laminated composite plates. *Department of Materials Science and Engineering, Massachusetts Institute of Technology, Cambridge*, pp. 1-17.

[10] Kant T. and Shiyekar S.M. (2012) "An assessment of a higher order theory for composite laminates subjected to thermal gradient" Journal of Composite Structures, Vol. 96, pp. 698–707.

[11] Mantari J.L., Oktem A.S. & Guedes Soares C. (2012). A new trigonometric shear deformation theory for isotropic, laminated composite and sandwich plates. *International Journal of Solids and Structures*, *49*, 43–53.

[12] Tornabene F., Liverani A. & Caligiana G. (2012). Laminated composite rectangular and annular plates: A GDQ solution for static analysis with a posteriori shear and normal stress recovery. *Composites: Part B, 43*, 1847–1872.

[13] Thai C.H., Ferreira A.J.M. & Bordas S.P.A. (2013). Isogeometric analysis of laminated composite and sandwich plates using a new inverse trigonometric shear deformation theory. *European Journal of Mechanics A/Solids, 43*, 89-108.

[14] Ghugal Y.M. & Kulkarni K.S. (2013). Thermal flexural analysis of cross-ply laminated plates using trigonometric shear deformation theory. *Latin American Journal of Solids and Structure, 10*, 1001–1023.

[15] Srinivasan S & Rao AK. (1970). Bending, vibration and buckling of simply supported thick orthotropic rectangular plates and laminates. *International Journal of Solids and Structures, 6*(11), 1463-1481.

[16] Pagano NJ. (1970). Exact solutions for rectangular bidirectional composites and sandwich plates. *Journal of Composite Materials, 4*(1), 20-34.

[17] M Karama, K S Afaq & S Mistou. (2009). A new theory for laminated composite plates. *Proc. IMechE, 223 Part L: J. Materials: Design and Applications*.

[18] Reddy JN. (2003). *Mechanics of laminated composite Plates and Shells*. (2nd ed.). Boca Raton London, New York: CRC Press.

[19] SD Kulkarni & S Kapuria. (2007). A new discrete Kirchhoff quadrilateral element based on the third-order theory for composite plates. *Computational Mechanics, 39*(3), 237-246.

[20] Sheikh AH & Chakrabarti A. (2003). A new plate bending element based on higher order shear deformation theory for the analysis of composite plates. *Finite Elements in Analysis and Design, 39*(9), 883-903.

[21] Chen W & Wu Z. (2005). A new higher-order shear deformation theory and refined beam element of composite laminates. *Acta MechSinica, 21*, 65-69.

[22] ABAQUS User's Manual. (2003). *ABAQUS Students Version 6.10*.

[23] Sahooand R & Singh B N. (2013). A new shear deformation theory for the static analysis of laminated composite and sandwich plates. *International Journal of Mechanical Sciences, 75*, 324–336.

[24] Kant, T & K. Swaminathan. (2002). Analytical solutions for the static analysis of laminated composite and sandwich plates based on a higher order refined theory. *Composite Structures, 56*, 329-344.

[25] Reddy J.N. & Liu C.F. (1985) A higher-order shear deformation theory of laminated elastic shells, *International Journal of Engineering Science, 23*(3), 319-330.