e-ISSN: 2250-0758 | p-ISSN: 2394-6962
Volume-10, Issue-3 (June 2020)
https://doi.org/10.31033/ijemr.10.3.2

Super Fair Dominating Set in the Cartesian Product of Graphs

Glenna T. Gemina[1] and Enrico L. Enriquez[2]
[1]Faculty, Saint Francis Academy, Balamban, Cebu, PHILIPPINES
[2]Faculty, Department of Computer, Information Science and Mathematics, University of San Carlos, PHILIPPINES

[2]Corresponding Author: enricolenriquez@yahoo.com

ABSTRACT

In this paper, we characterize the super fair dominating set in the Cartesian product of two graphs and give some important results.

Keywords**—** Fair Dominating Set, Super Dominating Set, Super Fair Dominating Set, Cartesian Product

I. INTRODUCTION

Domination in a graph has been a huge area of research in graph theory. Let G be a simple connected graph. A subset S of a vertex set $V(G)$ is a dominating set of G if, for every vertex $v \in V(G) \setminus S$, there exists a vertex $x \in S$ such that xv is an edge of G. The domination number $\gamma(G)$ of G is the smallest cardinality of a dominating set S of G. Domination in graph was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [1]. Some relate graph domination studies are found in [2,3,4,5,6,7,8,9,10,11,12,13,14].

One variant of domination in a graph is the fair domination in graphs [15]. A dominating subset S of $V(G)$ is a fair dominating set of G if all the vertices not in S are dominated by the same number of vertices from S, that is, $|N(u) \cap S| = |N(v) \cap S|$ for every two distinct vertices u and v from $V(G) \setminus S$ and a subset S of $V(G)$ is a k-fair dominating set in G if for every vertex $v \in V(G) \setminus S$, $|N(v) \cap S| = k$. The minimum cardinality of a fair dominating set of G, denoted by $\gamma_{fd}(G)$, is called the fair domination number of G. A fair dominating set of cardinality $\gamma_{fd}(G)$ is called γ_{fd}-set. A related paper on fair domination in graphs is found in [16,17]. Other variant of domination in a graph is the super dominating sets in graphs initiated by Lemanska et.al. [18]. A set $D \subset V(G)$ is called a super dominating set if for every vertex $u \in V(G) \setminus D$, there exists $v \in D$ such that $N_G(v) \cap (V(G) \setminus D) = \{u\}$. The super domination number of G is the minimum cardinality among all super dominating set in G denoted by $\gamma_{sp}(G)$. Variation of super domination in graphs can be read in [19,20,21,22].

A fair dominating set $S \subseteq V(G)$ is a super fair dominating set (or SFD-set) if for every $u \in V(G) \setminus S$, there exists $v \in S$ such that $N_G(v) \cap (V(G) \setminus S) = \{u\}$. The minimum cardinality of an SFD-set, denoted by $\gamma_{sfd}(G)$, is called the super fair domination number of G. The super fair dominating set was initiated by Enriquez [23]. In this paper, we extend the idea of super fair dominating set by characterizing the super fair dominating sets of the corona, lexicographic, and Cartesian product of two graphs. For general concepts we refer the reader to [24].

II. RESULTS

Remarks 2.1 *A super fair dominating set is a super dominating and a fair dominating set of a nontrivial graph G.*

Since the minimum super dominating set S of a nontrivial complete graph K_n is n - 1, it follows that $\gamma_{sfd}(K_n) = n - 1$. With this observation, the following remark holds.

Remark 2.2 *Let G be a nontrivial connected graph G of order n. Then* $1 \leq \gamma_{fd}(G) \leq \gamma_{sfd}(G) \leq n - 1$.

The Cartesian product $G \boxdot H$ of two graphs G and H is the graph with $V(G \boxdot H) = V(G) \times V(H)$ and $(u,u')(v,v') \in E(G \backslash sq \boxdot H)$ if and only if either $uv \in E(G)$ and $u' = v'$ or $u = v$ and $u'v' \in E(H)$. Note that if $C \subseteq V(G \times H)$, then the G-projection and H-projection of C are, respectively, the sets

$$C_G = \{u \in V(G): (u,b) \in C \text{ for some } b \in V(H)\} \text{ and}$$

$$C_H = \{v \in V(H): (a,v) \in C \text{ for some } a \in V(G)\}.$$

Lemma 2.3 *Let G and H be nontrivial connected graphs and S_H is a super fair dominating set of H. Then S is a super fair dominating set of $G \boxdot H$ if $S = V(G) \times S_H$.*

Proof: Suppose that $S = V(G) \times S_H$ where S_H is a super fair dominating set of H. Let $(x,y) \in V(G \boxdot H) \setminus S$. Then $x \in V(G)$ and $y \in V(H) \setminus S_H$. Since S_H is a super dominating set of H, there exists $y' \in S_H$ such that $N_H(y') \cap (V(H) \setminus S_H) = \{y\}$. This implies that $yy' \in E(H)$, that is, $(x,y)(x,y') \in E(G \boxdot H)$ with $(x,y') \in S$. Thus, for each $(x,y) \in V(G \boxdot H) \setminus S$, there exists $(x,y') \in S$ such that $N_{G \boxdot H}(x,y') \cap (V(G \boxdot H) \setminus S) = \{(x,y)\}$. Since (x,y) was arbitrarily

e-ISSN: 2250-0758 | p-ISSN: 2394-6962
Volume-10, Issue-3 (June 2020)
https://doi.org/10.31033/ijemr.10.3.2

chosen element of $V(G \boxdot H) \setminus S$, it follows that S is a super dominating set of $G \boxdot H$. Further, S_H is a fair dominating set of H implies that for every distinct elements $u, v \in V(H) \setminus S_H$, $|N_H(u) \cap S_H| = |N_H(v) \cap S_H|$. Thus, (x, u) and (x, v) are distinct elements of $V(G \boxdot H) \setminus S$ such that $|N_{G\boxdot H}\big((x,u)\big) \cap S| = |N_{G\boxdot H}\big((x,v)\big) \cap S|$. Since (x, u) and (x, v) are arbitrarily chosen elements of $V(G \boxdot H) \setminus S$, it follows that S is a fair dominating set of $G \boxdot H$. Accordingly, S is a super fair dominating set of $G \boxdot H$. ■

Lemma 2.4 *Let G and H be nontrivial connected graphs and S_G is a super fair dominating set of G. Then S is a super fair dominating set of $G \boxdot H$ if $S = S_G \times V(H)$.*

Proof: Suppose that $S = S_G \times V(H)$ where S_G is a super fair dominating set of H. Let $(x, y) \in V(G \boxdot H) \setminus S$. Then $x \in V(G) \setminus S_G$ and $y \in V(H)$. Since S_G is a super dominating set of G, there exists $x' \in S_G$ such that $N_G(x') \cap (V(G) \setminus S_G) = \{x\}$. This implies that $xx' \in E(G)$, that is, $(x, y)(x', y) \in E(G \boxdot H)$ with $(x', y) \in S$. Thus, for each $(x, y) \in V(G \boxdot H) \setminus S$, there exists $(x', y) \in S$ such that $N_{G\boxdot H}(x', y) \cap (V(G \boxdot H) \setminus S) = \{(x, y)\}$. Since (x, y) was arbitrarily chosen element of $V(G \boxdot H) \setminus S$, it follows that S is a super dominating set of $G \boxdot H$. Further, S_G is a fair dominating set of G implies that for every distinct elements $u, v \in V(G) \setminus S_{G,}$ $|N_G(u) \cap S_G| = |N_G(v) \cap S_G|$. Thus, (u, y) and (v, y) are distinct elements of $V(G \boxdot H) \setminus S$ such that $|N_{G\boxdot H}\big((u,y)\big) \cap S| = |N_{G\boxdot H}\big((v,y)\big) \cap S|$. Since (u, y) and (v, y) are arbitrarily chosen elements of $V(G \boxdot H) \setminus S$, it follows that S is a fair dominating set of $G \boxdot H$. Accordingly, S is a super fair dominating set of $G \boxdot H$. ■

Lemma 2.5 *Let G and H be nontrivial connected graphs. Then S is a super fair dominating set of $G \boxdot H$ if $S =$ $(V(G) \times S\_H\ ) \cup\ (S_G \times (V(H) \setminus S_H))$ where S_G and S_H are super fair dominating sets of G and H respectively.*

Proof: Suppose that $S = (V(G) \times S_H) \cup (S_G \times (V(H) \setminus S_H))$ where S_G and S_H are super fair dominating sets of G and H respectively. Let $(x, y) \in V(G \boxdot H) \setminus S$. Then $x \in V(G) \setminus S_G$ and $y \in V(H) \setminus S_H$.

If S_G is a super dominating set of G, then there exists $x' \in S_G$ such that $N_G(x') \cap (V(G) \setminus S_G) = \{x\}$. This implies that $xx' \in E(G)$, that is, $(x, y)(x', y) \in E(G \boxdot H)$ with $(x', y) \in S$. Thus, for each $(x, y) \in V(G \boxdot H) \setminus S$, there exists $(x', y) \in S$ such that $N_{G\boxdot H}(x', y) \cap (V(G \boxdot H) \setminus S) = \{(x, y)\}$. If S_H is a super dominating set of H, then there exists $y' \in S_H$ such that $N_H(y') \cap (V(H) \setminus S_H) = \{y\}$. This implies that $yy' \in E(H)$, that is, $(x, y)(x, y') \in E(G \boxdot H)$ with $(x, y') \in S$. Thus, for each $(x, y) \in V(G \boxdot H) \setminus S$, there exists $(x, y') \in S$ such that $N_{G\boxdot H}(x, y') \cap (V(G \boxdot H) \setminus S) = \{(x, y)\}$. In either cases, S is a super dominating set of $G \boxdot H$.

Further, if S_G is a fair dominating set of G, then for every distinct elements $u, v \in V(G) \setminus S_G$, $|N_G(u) \cap S_G| = |N_G(v) \cap S_G|$. Thus, (u, y) and (v, y) are distinct elements of $V(G \boxdot H) \setminus S$ such that $|N_{G\boxdot H}\big((u,y)\big) \cap S| = |N_{G\boxdot H}\big((v,y)\big) \cap S|$. Finally, if S_H is a fair dominating set of H, then for every distinct element $y, y' \in V(H) \setminus S_H$, $|N_H(y) \cap S_H| = |N_H(y') \cap S_H|$. Thus, (u, y) and (u, y') are distinct elements of $V(G \boxdot H) \setminus S$ such that

$$|N_{G\boxdot H}\big((u,y)\big) \cap S| = |N_{G\boxdot H}\big((u,y')\big) \cap S|.$$

In either cases, S is a fair dominating set of $G \boxdot H$. Accordingly, S is a super fair dominating set of $G \boxdot H$. ■

Lemma 2.6 *Let G and H be nontrivial connected graphs. Then S is a super fair dominating set of $G \boxdot H$ if $S = \big(S_G \times V(H)\big) \cup ((V(G) \setminus S_G) \times S_H)$ where S_G and S_H are super dominating sets of G and H respectively.*

Proof: Suppose that $S = \big(S_G \times V(H)\big) \cup ((V(G) \setminus S_G) \times S_H)$ where S_G and S_H are super dominating sets of G and H respectively. Let $(x, y) \in V(G \boxdot H) \setminus S$. Then $x \in V(G) \setminus S_G$ and $y \in V(H) \setminus S_H$.

If S_G is a super dominating set of \$$G$\$, then there exists $x' \in S_G$ such that $N_G(x') \cap (V(G) \setminus S_G) = \{x\}$. This implies that $xx' \in E(G)$, that is, $(x, y)(x', y) \in E(G \boxdot H)$ with $(x', y) \in S$. Thus, for each $(x, y) \in V(G \boxdot H) \setminus S$, there exists $(x', y) \in S$ such that $N_{G\boxdot H}(x', y) \cap (V(G \boxdot H) \setminus S) = \{(x, y)\}$. If S_H is a super dominating set of H, then there exists $y' \in S_H$ such that $N_H(y') \cap (V(H) \setminus S_H) = \{y\}$. This implies that $yy' \in E(H)$, that is, $(x, y)(x, y') \in E(G \boxdot H)$ with $(x, y') \in S$. Thus, for each $(x, y) \in V(G \boxdot H) \setminus S$, there exists $(x, y') \in S$ such that $N_{G\boxdot H}(x, y') \cap (V(G \boxdot H) \setminus S) = \{(x, y)\}$. In either cases, S is a super dominating set of $G \boxdot H$.

Further, if S_G is a fair dominating set of G, then for every distinct elements $u, v \in V(G) \setminus S_G$, $|N_G(u) \cap S_G| = |N_G(v) \cap S_G|$. Thus, (u, y) and (v, y) are distinct elements of $V(G \boxdot H) \setminus S$ such that $|N_{G\boxdot H}\big((u,y)\big) \cap S| = |N_{G\boxdot H}\big((v,y)\big) \cap S|$. Finally, if S_H is a fair dominating set of H, then for every distinct elements $y, y' \in V(H) \setminus S_H$, $|N_H(y) \cap S_H| = |N_H(y') \cap S_H|$. Thus, (u, y) and (u, y') are distinct elements of $V(G \boxdot H) \setminus S$ such that $|N_{G\boxdot H}\big((u,y)\big) \cap S| = |N_{G\boxdot H}\big((u,y')\big) \cap S|$. In either cases, S is a fair dominating set of $G \boxdot H$.

Accordingly, S is a super fair dominating set of $G \boxdot H$. ■

The next result is the characterization of the super fair dominating set in the Cartesian product of two graphs.

e-ISSN: 2250-0758 | p-ISSN: 2394-6962
Volume-10, Issue-3 (June 2020)
https://doi.org/10.31033/ijemr.10.3.2

Theorem 2.7 *Let G and H be nontrivial connected graphs. Then a proper subset S of $V(G \boxdot H)$ is a super fair dominating set of $G \boxdot H$ if and only if one of the following statements is satisfied.*

(i) *$S = V(G) \times S_H$ where S_H is a super fair dominating set of H.*

(ii) *$S = S_G \times V(H)$ where S_G is a super fair dominating set of G.*

(iii) *$S = (V(G) \times S_H) \cup (S_G \times (V(H) \setminus S_H))$ where S_G and S_H are super fair dominating sets of G and H respectively.*

(iv) *$S = (S_G \times V(H)) \cup ((V(G) \setminus S_G) \times S_H)$ where S_G and S_H are super fair dominating sets of G and H respectively.*

Proof: Suppose that a proper subset S of $V(G \boxdot H)$ is a super fair dominating set of $G\backslash square \boxdot H$.

Let $S_H \subset V(H)$. Since S is a fair dominating set, for every distinct elements (x, y) and (x, y') of $V(G \boxdot H\backslash S$, |NG⊡Hx,y∩S|=|NG⊡Hx,y′∩S|. Thus, for every distinct elements y and y' of $V(H) \setminus S_H$, $|N_H(y) \cap S_H| = |N_H(y') \cap S_H|$. By definition, S_H is a fair dominating set of H. Since S is a super dominating set, for every $(x, y) \in V(G \boxdot H) \setminus S$, there exists $(x, y') \in S$ such that

$$N_{G\boxdot H}\{G((x, y')) \cap (V(G \boxdot H) \setminus S) = \{(x, y)\}.$$

Thus, for every $y \in V(H) \setminus S_H$, there exists $y' \in S_H$ such that $N_H(y') \cap (V(H) \setminus S_H) = \{y\}$. By definition, S_H is a super dominating set of H. Hence, S_H is a super fair dominating set of H and $V(G) \times S_H$ must be a super fair dominating set of $G \boxdot H$. Set $S = V(G) \times S_H$. This proves statement (i).

Let $S_G \subset V(G)$. Since S is a fair dominating set, for every distinct elements (u, y) and (v, y) of $V(G \boxdot H\backslash S\$$, |NG⊡Hu,y∩S|=|NG⊡Hv,y∩S|. Thus, for every distinct elements u and v of $V(G) \setminus S_G$, $|N_G(u) \cap S_G| = |N_G(v) \cap S_G|$. By definition, S_G is a fair dominating set of G. Since S is a super dominating set, for every $(x, y) \in V(G \boxdot H) \setminus S$, there exists $(x', y) \in S$ such that $N_{G\boxdot H}((x', y)) \cap (V(G \boxdot H) \setminus S) = \{(x, y)\}$. Thus, for every $x \in V(G) \setminus S_G$, there exists $x' \in S_G$ such that $N_G(x') \cap (V(G) \setminus S_G) = \{x\}$. By definition, S_G is a super dominating set of G. Hence, S_G is a super fair dominating set of G and $S_G \times V(H)$ must be a super fair dominating set of $G \boxdot H$. Set $S = S_G \times V(H)$. This proves statement (ii).

By statement (i), $V(G) \times S_H$ is a super fair dominating set of $G \boxdot H$ with S_H is a super fair dominating set of H. By statement (ii), $S_G \times V(H)$ is a super fair dominating set of $G \boxdot H$ with S_G is a super fair dominating set of G. Suppose that $C = (V(G) \times S_H) \cup (S_G \times V(H))$. Let $(x, y) \in V(G \boxdot H) \setminus C$. Then $x \in V(G) \setminus S_G$ and $y \in V(H) \setminus S_H$. Since S_G is a super dominating set of G, there exists $x' \in S_G$ such that $N_G(x') \cap V(G) \setminus S_G = \{x\}$. Since S_H is a super dominating set of H, there exists $y' \in S_H$ such that $N_H(y') \cap V(H) \setminus S_H = \{y\}$. Thus, for every $(x, y) \in V(G \boxdot H) \setminus C$ there exists (x', y) such that $N_{G\boxdot H}((x', y)) \cap (V(G \boxdot H) \setminus C) = \{(x, y)\}\$$ or there exists (x, y') such that $N_{G\boxdot H}((x, y')) \cap (V(G \boxdot H) \setminus C) = \{(x, y)\}$. This implies that C is a super dominating set of $G \boxdot H$.

Further, since S_G is a fair dominating set of G, for every distinct elements $x, u \in V(G) \setminus S_G$, $|N_G(x) \cap S_G| = |N_G(u) \cap S_G|$ and since S_H is a fair dominating set of H, for every distinct elements $y, v \in V(H) \setminus S_H$, $|N_H(y) \cap S_H| = |N_H(v) \cap S_H|$. Thus, for every distinct elements $(x, y), (x, v) \in V(G \boxdot H) \setminus C$, $|N_{G\boxdot H}((x, y)) \cap C| = |N_{G\boxdot H}((x, v)) \cap C|$ or for every distinct elements $(x, y), (u, y) \in V(G \boxdot H) \setminus C$, $|N_{G\boxdot H}((x, y)) \cap C| = |N_{G\boxdot H}((u, y)) \cap C|$. This implies that C is a fair dominating set of $G \boxdot H$. Accordingly C is a super fair dominating set of $G \boxdot H$.

Since $C = (V(G) \times S_H) \cup (S_G \times V(H)) = (V(G) \times S_H) \cup (S_G \times V(H) \setminus S_H)$ is clear, set $S = (V(G) \times S_H) \cup (S_G \times V(H) \setminus S_H)$. This proves statement (iii). Similarly, statement (iv) follows.

For the converse, suppose that statement (i) is satisfied. Then $S = V(G) \times S_H$ where S_H is a super fair dominating set of H. By Lemma 2.3, S is a super fair dominating set of $G \boxdot H$. Suppose that statement (ii) is satisfied. Then $S = S_G \times V(H)$ where S_G is a super fair dominating set of G. By Lemma 2.4, S is a super fair dominating set of $G \boxdot H$. Suppose that (iii) is satisfied. Then $S = (V(G) \times S_H) \cup (S_G \times (V(H) \setminus S_H))$ where S_G and S_H are super fair dominating sets of G and H respectively. By Lemma 2.5, S is a super fair dominating set of $G \boxdot H$. If statement (iv) holds, then $S = (S_G \times V(H)) \cup ((V(G) \setminus S_G) \times S_H)$ where S_G and S_H are super fair dominating sets of G and H respectively.

By Lemma 2.5, S is a super fair dominating set of $G \boxdot H$. This complete that proofs. ■

The following result is an immediate consequence of Theorem 2.7.

Corollary 2.8 *Let G and H be nontrivial connected graphs of orders m and n respectively. Then*

$$\gamma_{sfd}(G \boxdot H) = \min\{m \cdot \gamma_{sfd}(H), \gamma_{sfd}(G) \cdot n\}.$$

Proof: Suppose that $S = V(G) \times S_H$ where S_H is a super fair dominating set of H. Then S is a super fair dominating set of $G \boxdot H$ by Theorem 2.7. This implies that

$$\begin{aligned} \gamma_{sfd}(G \boxdot H) &\leq |S| \\ &= |V(G) \times S_H| \\ &= |V(G)| \cdot |S_H| \end{aligned}$$

e-ISSN: 2250-0758 | p-ISSN: 2394-6962
Volume-10, Issue-3 (June 2020)

https://doi.org/10.31033/ijemr.10.3.2

$= m \cdot |S_H|$

for all super fair dominating set S_H. Hence,

$$\gamma_{sfd}(G \boxdot H) \leq m \cdot \gamma_{sfd}(H).$$

Suppose that $S = S_G \times V(H)$ where S_G is a super fair dominating set of G. Then S is a super fair dominating set of $G \boxdot H$ by Theorem 2.7. This implies that

$$\begin{aligned}\gamma_{sfd}(G \boxdot H) &\leq |S| \\ &= |S_G \times V(H)| \\ &= |S_G| \cdot |V(H)| \\ &= |S_G| \cdot n\end{aligned}$$

for all super fair dominating set S_G .

Hence, $\gamma_{sfd}(G \boxdot H) \leq \gamma_{sfd}(G) \cdot n$. Thus,

$$\gamma_{sfd}(G \boxdot H) \leq \min\{m \cdot \gamma_{sfd}(H), \gamma_{sfd}(G) \cdot n\}.$$

Now, let S^o be a γ_{sfd}-set of $G \boxdot H$. Then $|S^o| = \min\{|S|: S$ is a super fair dominating set of $G \boxdot H\}$.

Consider the following cases.

Case 1. Suppose that $|S^o| \leq m \cdot \gamma_{sfd}(H)$.

If $|S^o| = m \cdot \gamma_{sfd}(H)$, then

$$|S^o| = \min\{m \cdot \gamma_{sdf}(H), \gamma_{sfd}(G) \cdot n\}.$$

If $|S^o| < m \cdot \gamma_{sfd}(H)$, then consider the next case.

Case 2. Consider that $|S^o| \leq \gamma_{sfd}(G) \cdot n$.

If $|S^o| = \gamma_{sfd}(G) \cdot n$, then

$$|S^o| = \min\{m \cdot \gamma_{sfd}(H), \gamma_{sfd}(G) \cdot n\}.$$

If $|S^o| < \gamma_{sfd}(G) \cdot n$, then consider the next case.

Case 3. Consider that $|S^o| < m \cdot \gamma_{sfd}(H)$ and $|S^o| < \gamma_{sfd}(G) \cdot n$. Then $|S^o| < \min\{m \cdot \gamma_{sfd}(H), \gamma_{sfd}(G) \cdot n\}$.

Suppose that $|S^o| = (m-1)\gamma_{sfd}(H)$. Let $x \in V(G) \setminus S_G$ where $S_G \subset V(G)$ and let $a \in V(H) \setminus S_H$ where S_H is a super fair dominating set of H. Then $(x,a) \in V(G \boxdot H) \setminus S^o$ and $(x,a)(u,v) \notin E(G \boxdot H)$ for all $(u,v) \in S^o$. Thus, S^o is not a dominating set of $G \boxdot H$ contradict to the fact that S^o is a super fair dominating set of $G \boxdot H$. Hence, $|S^o| \neq (m-1)\gamma_{sfd}(H)$. Similarly, if S_H is not a dominating set of H, then S^o is not a dominating set of $G \boxdot H$, a contradiction. Moreover, using the same arguments, if $|S^o| = \gamma_{sfd}(G)(n-1)$, then S^o is not a dominating set of $G \boxdot H$, a contradiction. Hence, $|S^o| \neq \gamma_{sfd}(G)(n-1)$. If S_G is not a dominating set of G, then S^o is not a dominating set of $G \boxdot H$, a contradiction. Thus, $|S^o|$ is not lesser than $\{m \cdot \gamma_{sfd}(H), \gamma_{sfd}(G) \cdot n\}$, that is,

$$|S^o| \geq \{m \cdot \gamma_{sfd}(H), \gamma_{sfd}(G) \cdot n\}.$$

Consequently, $|S^o| = min\{m \cdot \gamma_{sfd}(H), \gamma_{sfd}(G) \cdot n\}$

Since S^o is a γ_{sfd}-set of $G \boxdot H$, it follows that

$$\gamma_{sfd}(G \boxdot H) = min\{m \cdot \gamma_{sfd}(H), \gamma_{sfd}(G) \cdot n\}. \blacksquare$$

V. CONCLUSION

In this paper, we extend the concept of the super fair domination in graphs. The super fair domination in the Cartesian product of two connected graphs were characterized. The exact super fair domination number of the Cartesian product of two connected graphs was computed. This study will motivate math research enthusiasts to work on super fair dominating set of other possible binary operation of two graphs. Other parameters involving super fair domination in graphs may also be explored. Finally, the characterization of a super fair domination in graphs and its bounds is also a possible extension of this study.

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e-ISSN: 2250-0758 | p-ISSN: 2394-6962
Volume-10, Issue-3 (June 2020)
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