

Modal Space Controller for Hydraulically Driven Six Degree of Freedom Parallel Manipulator

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ABSTRACT

This paper presents the Modal space decoupled control for a hydraulically driven parallel mechanism has been presented. The approach is based on singular values decomposition to the properties of joint-space inverse mass matrix, and mapping of the control and feedback variables from the joint space to the decoupling modal space. The method transformed highly coupled six-input six-output dynamics into six independent single-input single-output (SISO) 1 DOF hydraulically driven mechanical systems. The novelty in this method is that the signals including control errors, control outputs and pressure feedbacks are transformed into decoupled modal space and also the proportional gains and dynamic pressure feedback are tuned in modal space. The results indicate that the conventional controller can only attenuate the resonance peaks of the lower eigenfrequencies of six rigid modes properly, and the peaking points of other relative higher eigenfrequencies are over damped. The further results show that it is very effective to design and tune the system in modal space and that the bandwidth increased substantially except surge (x) and sway (y) motions, each degree of freedom can be almost tuned independently and their bandwidths can be increased near to the undamped eigenfrequencies.

Keywords-- Hydraulically Driven, 6 DOF Parallel Mechanism, Modal Space, Dynamic Pressure Feedback

I. INTRODUCTION

Modal space control has in the recent time become popular as a control strategy in linear structural dynamics and related fields like aero elasticity and active vibration control and robotics. This type of control concept originated from process control problems, which were summarized in Porter and Crossley [1]. Gould and Muarray-Lasso intended the modal control concepts to structure systems [2]. Meirovitch [3] developed an independent modal space control (IMSC) method for controlling a single mode of a distributed mass body. In IMSC, the equations of motion for the structure are decoupled using modal analysis and then modal control forces are determined by minimizing a performance index. Baz and Poh [4] in their study performed a numerical

study on controlling the vibrations of a beam instrumented with piezoelectric patches, using IMSC. They modified the IMSC to control multiple modes and called the result modified independent modal space control (MIMSC). Baz and Poh [5] experimentally verified the theoretical developments by controlling the first two modes of a cantilevered beam.

Other notable works can be found in Anthonis [6]. He applied modal space decoupling control to control the active suspension of a spray boom. The two hydraulic actuators of the active suspension are placed symmetrically on the spray boom. Because the spray boom itself is also symmetrical, the two excitations can be transformed to a translational and rotational excitation. This modal transformation yields perfect decoupling in the frequency range where only the rigid body modes determine the vibration. Lauwerys, Swevers and Sas [6] used a similar approach to control an active suspension of a car. The motion of the car body is transformed in heave, roll and pitch motions which are decoupled. Lin and Yu [7] used modal decoupling to suppress the vibration of a rotor.

The popularity of modal space control is due to its numerous benefits. One of the most important advantages is its ability to reduce the order of a system by choosing the most important modes of the system. Also, the natural modes decouple the structural system and further simplify the system analysis, no optimization is required and numerically stable methods exist to determine the modal vectors. In addition, a tremendous insight is gained into the characteristics and behavior of the system [8].

While significant progress has been made in the field of modal space control concept, the application of modal space to the control of hydraulically driven 6 DOF motion simulation platform has not been fully exploited. In this study the application of modal space control strategy, which is receiving intense interest in the control of multi-degree-of-freedom (MDOF) systems and has been successively applied to a wide variety of practical problems, is examined. It is the purpose of this work to extend the modal space control to 6 DOF motion simulation platform by using modal space decoupling control strategy (MSDC). The methodology is based on exploitation of the properties of the joint space inverse

mass matrix. Through a mapping of the control and feedback variables, from the joint space to the decoupling modal space, the highly coupled MDOF dynamics is transformed into six independent single-input single-output (SISO) 1 DOF hydraulically driven mechanical system.

II. SIX DEGREES OF FREEDOM PARALLEL MANIPULATOR (PM) DYNAMICS

The 6 DOF PM is a parallel mechanism that consists of a rigid body top plate, or mobile plate, connected to a fixed base plate and is defined by at least

three stationary points on the grounded base connected to six independent kinematic legs. The six legs of the 6 DOF PM are connected to both the base plate and the top plate by universal joints in parallel located at both ends of each leg. The legs are designed with an upper body and lower body that can be adjusted, allowing each leg to be varied in length. Each leg subsystem contains two bodies connected together with a cylindrical joint. The position and orientation of the mobile platform varies depending on the lengths to which the six legs are adjusted. The overall system has six degrees of freedom. The mechanism is depicted pictorially in figure 1.

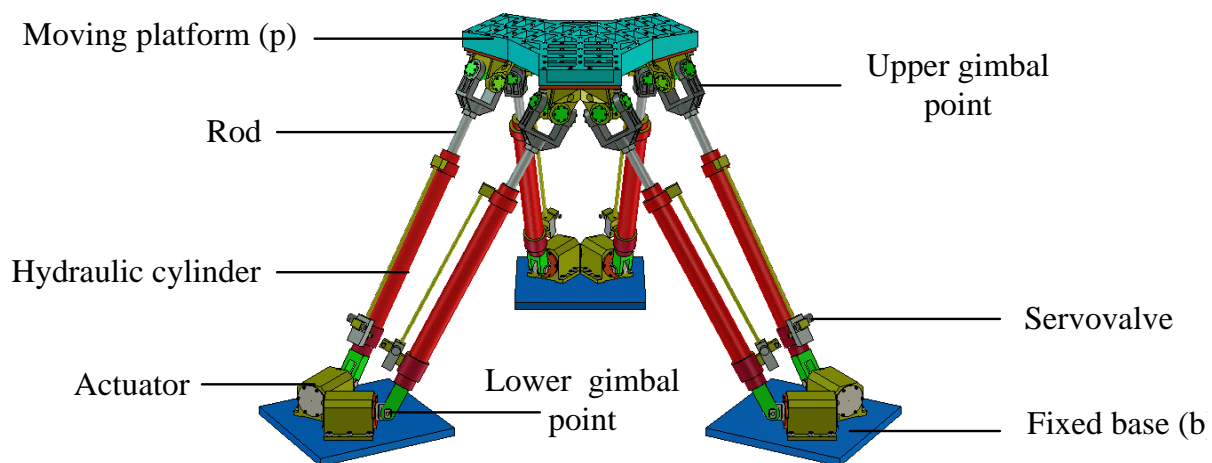


Figure 1: Hydraulically Driven 6 DOF Parallel Manipulator

III. SIX DEGREES OF FREEDOM PM KINEMATICS

The kinematics analysis that establishes the relationship between the lengths of the six actuators and the position of the moving platform will be presented. There are two main axis systems used to describe the motion: body-fixed and earth-fixed coordinates. The calculations are done in the body-fixed axis to simplify the equations; the results are then transformed to the earth-fixed coordinate system for proper updating of position and orientation. The earth-fixed system is considered an inertial reference frame, neglecting the rotational velocity of the earth and conforming to Newton's laws of motion. A flat earth assumption is made, which essentially aligns the acceleration of gravity along the vertical earth-fixed

system.

In figure 3 a schematic drawing of the coordinate system is shown, addressing the geometric relations of the system with coordinates of 6 DOF PM. The six joints are on the center A_i ($i = 1, 2, \dots, 6$), located in the circle with radius r_a , and denoted by A_1, A_3, A_5 and A_2, A_4, A_6 .

The center of the six joints is connected to form a hexagon. Similarly, six of the attachment points B_i ($i = 1, 2, \dots, 6$), located in the circle of radius r_b , and B_1, B_3, B_2 and B_4, B_5, B_6 , respectively constitute two equilateral triangles. The connection of the upper and lower attachment points form a platform rotated 180 degrees with respect to each other. The motion of the moving platform is generated by actuating the cylindrical joints which vary the lengths of the legs, l_i , $i = 1, \dots, 6$.

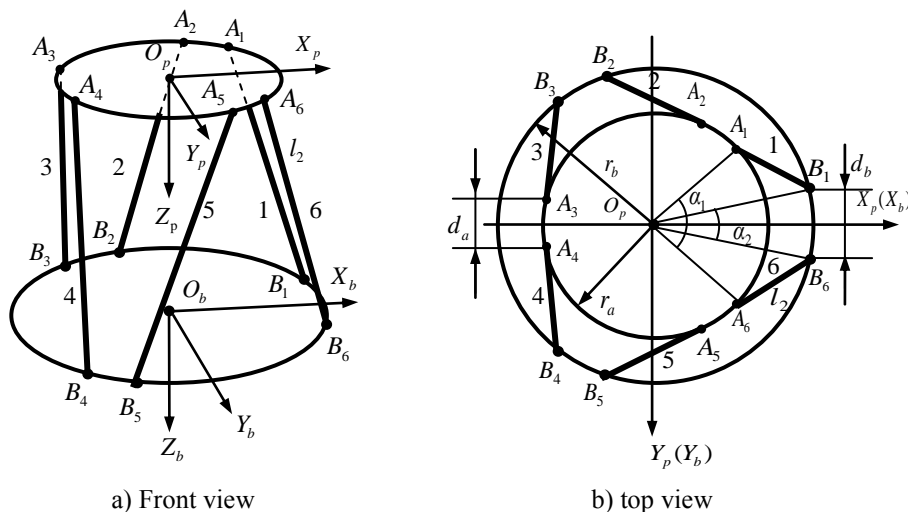


Figure 3: Schematic diagram of coordinate system of 6-DOF PM

So, trajectory of the center point of moving platform is adjusted by using these variables. For modeling of the mechanism, a base reference frame $\{B\}$ (O_B, X_B, Y_B, Z_B) is defined as shown in figure 3. A second frame $\{p\}$ (O_p, X_p, Y_p, Z_p) is attached to the center of the moving platform, O_p and the points linking the legs to the moving platform are noted as $A_i, i = 1 \dots 6$, and each leg is attached to the base platform at the point $B_i, i = 1 \dots 6$. At neutral pose the body axes $\{p\}$ attached to the moving platform are parallel to and coincide with the inertial frame $\{B\}$ fixed to the base with its origin at the geometric centre of the base platform. Kinematics and dynamics analysis of the 6 DOF parallel mechanisms has been well established

and can be found in several literatures [9-13].

3.1 Control Design

The structure of the proposed modal space controller is similar to the conventional joint space control as shown in figure 4. However, the difference is that the signals including control errors, control outputs and pressure difference feedbacks are transformed into decoupled modal space, so that the coupling effects are fully considered and compensated. Thus, the proportional gains and dynamic pressure feedback functions can be tuned independently in modal space. Also, the bandwidth can be raised near to the undamped eigenfrequencies.

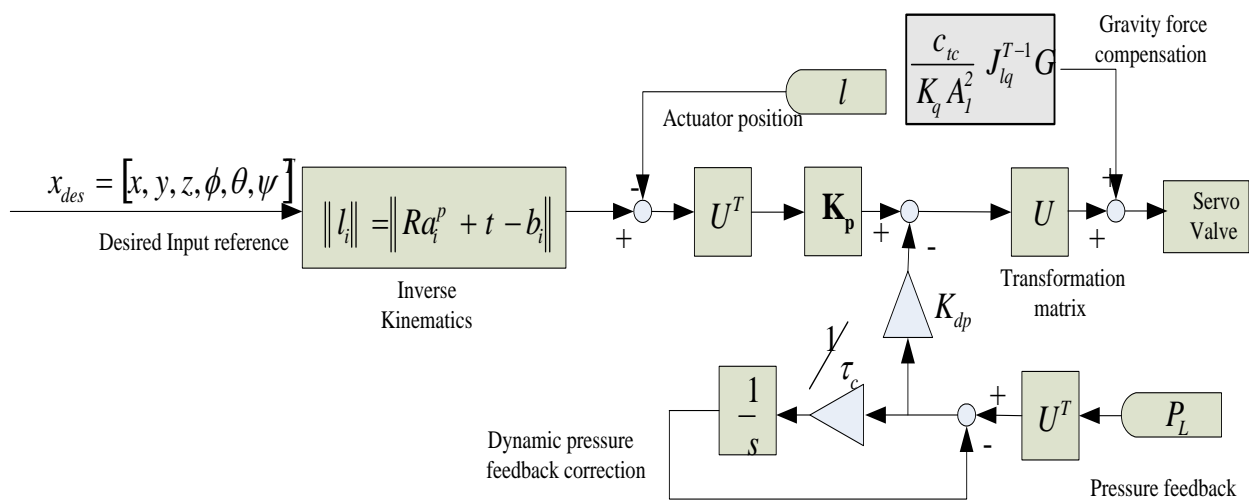


Figure 4: Structure of the modal space decoupled controller

Based on this model, the control inputs of the servo valves can be developed. It can be expressed as:

$$\begin{aligned} \bar{\dot{i}}_i &= U \text{diag}([k_{a,1} \quad k_{a,2} \quad \dots \quad k_{a,6}]^T) U^T e \\ &+ U \text{diag} \left(\left[k_{dp,1} \frac{\tau_{c,1} s}{\tau_{c,1} s + 1} \quad k_{dp,2} \frac{\tau_{c,2} s}{\tau_{c,2} s + 1} \quad \dots \quad k_{dp,6} \frac{\tau_{c,6} s}{\tau_{c,6} s + 1} \right]^T \right) U^T P_L \\ &+ \frac{c_{tc}}{K_q A_1^2} J_{lq}^{T-1} G \end{aligned} \tag{1}$$

$$e = \bar{l}_{com} - \bar{l} \tag{2}$$

Where e — position error (m),
 \bar{l}_{com} — actuator length command in terms of inverse kinematics (m).

And each element of $\bar{\dot{i}}_i$ can be give as

$$\bar{\dot{i}}_i = \sum_{i=1}^6 \left(\left(\sum_{k=1}^6 U_{ik} U_{jk} k_{a,k} \right) e_i + \left(\sum_{k=1}^6 U_{ik} U_{jk} k_{dp,k} \frac{\tau_{c,k} s}{\tau_{c,k} s + 1} \right) P_{L_i} + \frac{c_{tc}}{K_q A_1^2} J_{ij} G_i \right) \tag{3}$$

Where e — e_i (m),
 U_i — (U_{jk}) ,
 P_L — P_{L_i} (Pa),
 J_{lq}^{T-1} — (J_{ij}) ,
 G — (G_i) (m/s²).

When $k_{a,1} = k_{a,2} = \dots k_{a,6}$ and, $k_{dp,1} = k_{dp,2} = \dots k_{dp,6}$ and unitary orthogonal matrix, U equals one, then, the modal space controller degenerates into the conventional joint space controller.

IV. RESULTS AND DISCUSSION

In this section, a particular test case representing typical physical conditions are designed to demonstrate the performance and feasibility of the decoupling control strategy by integrating and testing our methodologies, and numerical simulation results from the PM is realized in the Matlab/Simulink environment. A comparison was being made between the modal space controller and the

conventional controller to discuss the effectiveness or otherwise of both controllers.

The evaluation of control performance of the 6 DOF PM as applicable to flight motion simulators, most often deals with the describing function. The describing function can be in the form of measuring the frequency response such that the magnitude of the response is flat. These description functions enable the characterization of some of the parameters which are considered most important in control such as bandwidth, damping and interaction effects in multivariable system [14].

The results of Bode diagrams of the closed loop system from the desired to actual positions applying the modal space controller are shown in figures 5 and 6 respectively.

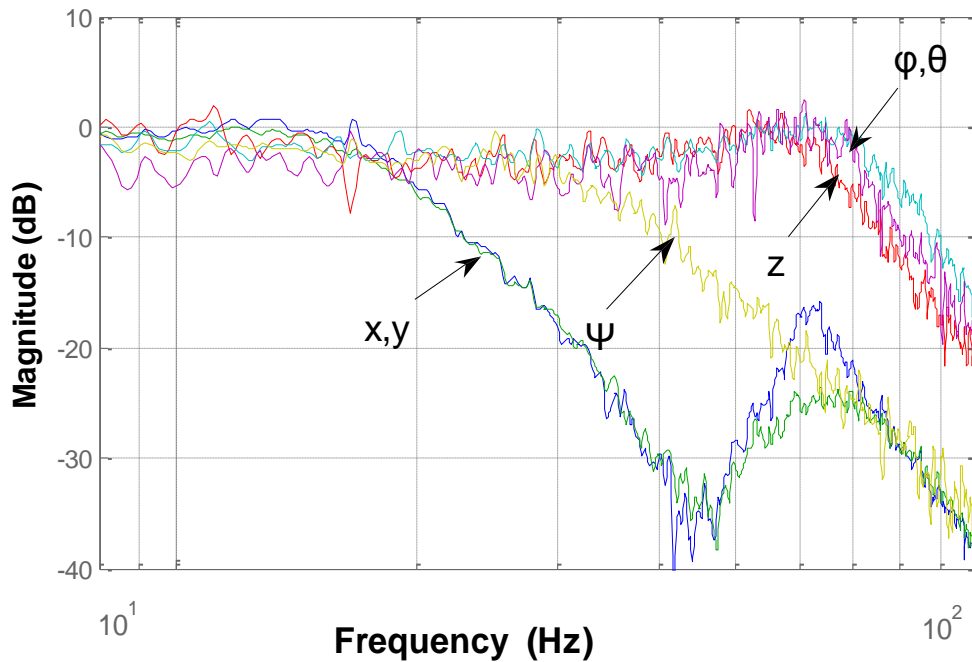


Figure 5: Simulation result of modal space controlled motion system frequency response

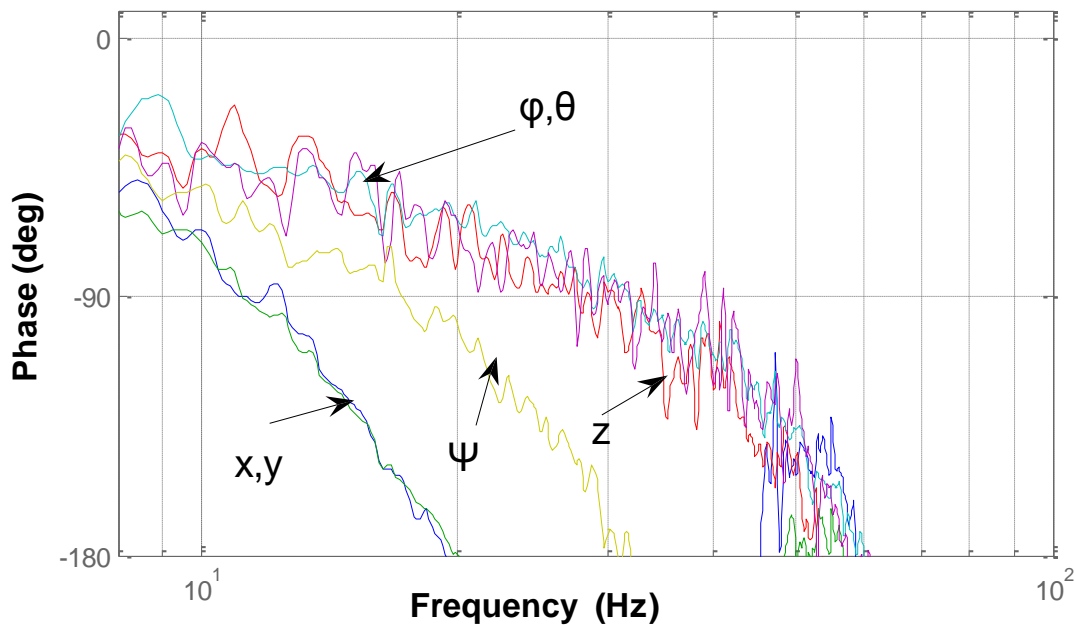


Figure 6: Simulation result of modal space controlled motion system frequency response

There are several characteristics of the system that can be read directly from this Bode plot. For example, in figures 5 and 6, the -3dB point can be found around 71.2Hz for pitch (R_y) and roll (R_x), 66.7Hz for heave (z), 29.4Hz for yaw. In surge(x) and sway (y) the bandwidth is just only 18.1Hz which is the lowest compared to other directions. The -90 deg bandwidth can be found at 28.7Hz

for pitch (R_y) and roll (R_x), 27.7Hz for heave (z), 17.2Hz for yaw (R_z), 11.2Hz for surge(x) and sway(y). Highest bandwidths are attained with pitch (R_y), roll (R_x) and lowest in surge(x) and sway(y). All responses are reasonably flat without peaking above +3dB. As mentioned earlier, our analysis and comparing is based on the performance of the controllers on hydraulically driven

6 DOF PM as it relates to flight motion simulator. The results obtained from modal space control strategy are compared with conventional joint space controller for the

dynamic responses of the closed loop system. The frequency response for the conventional joint space controller is given in Figures 7 and 8.

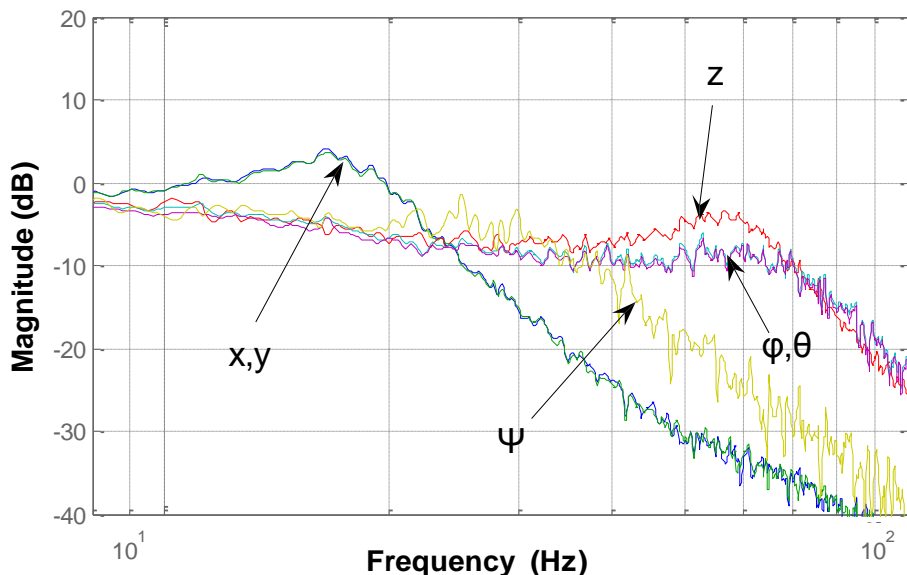


Figure 7: Simulation result of conventional PID controlled motion system frequency response

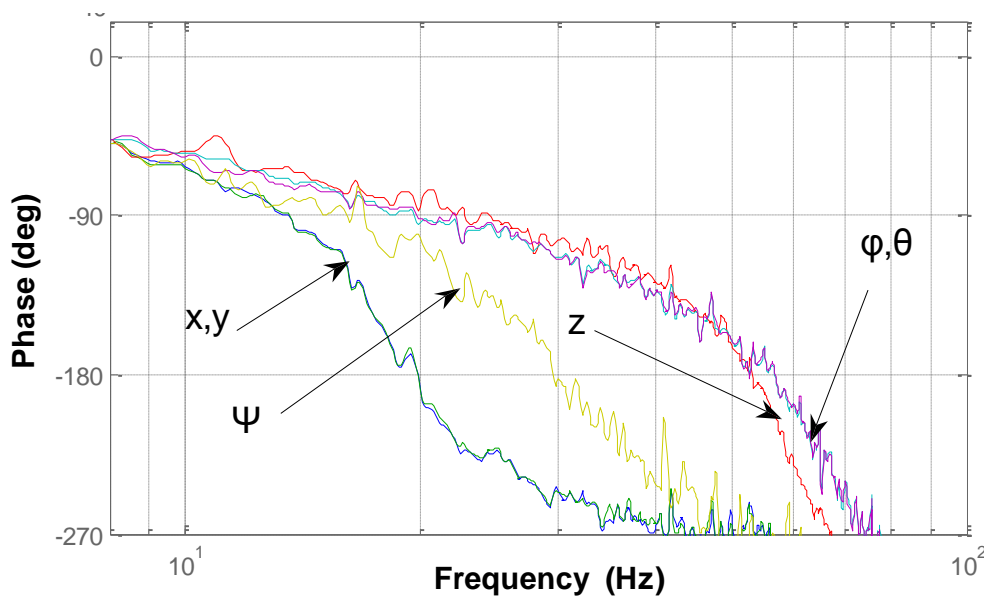


Figure 8: Simulation result of conventional PID controlled motion system frequency response

Because there is no compensation of coupling effects for the different natural frequencies, the responses of surge (x) and sway (y), demonstrate peaks much more than others (such as pitch (Ry) and roll (Rx), which are over damped). Except for surge and pitch, the bandwidth is as low as 8.4Hz.

Moreover, a compromise has to be found between peaking (up till 3dB) of the lowest bandwidth loops of

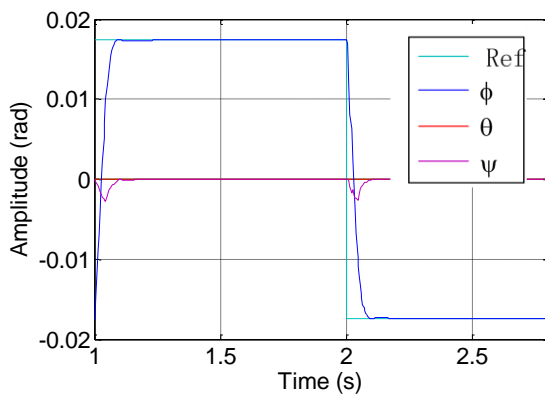
surge (x) and sway (y) and the over damped response of pitch (Ry) and roll (Rx). For this control structure, the control action is being limited by the lowest eigenfrequencies of the system, therefore, the maximum attainable bandwidth is 8.5Hz for both roll (Rx) and pitch (Ry). With the conventional controller the attainable bandwidth for x and y is 21.8Hz, which is higher than with modal space controller which is only 18.1Hz.

Nevertheless, this does not suggest that the dynamic performance along these degrees of freedom is deteriorated using modal space controller.

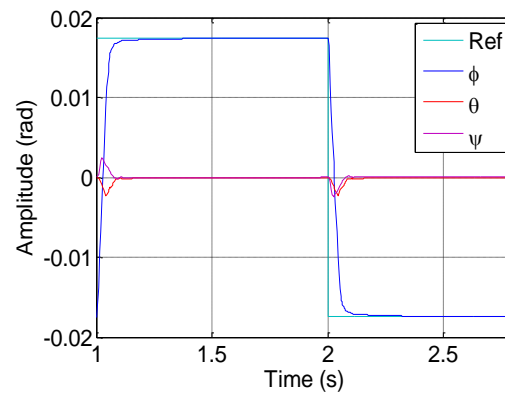
As can be observed in figure 9, the conventional control bandwidth is achieved at the expense of higher overshoot and poor performances of other degrees of freedom, however, a high percentage of overshoot is an undesirable phenomenon in flight motion simulator. Moreover, the bandwidth of the modal space controller can be tuned to a higher level, as the same as the conventional control, but that will not meet the requirements of a flight simulator motion system in which flat frequency response is preferable [15]. In addition, with the modal space controller, obviously each degree of freedom can be almost

tuned independently, and it is possible to raise their bandwidths near to the undamped eigenfrequencies. Comparing figure 5 and figure 6 we can see that the bandwidth of roll and pitch increased from 8.5Hz to 71.1Hz, yaw is raised from 8.4Hz to 29.4Hz, and heave increased from 9.2Hz to 66.7Hz. The comparison of two control strategies indicates that the modal space control performed better than the conventional.

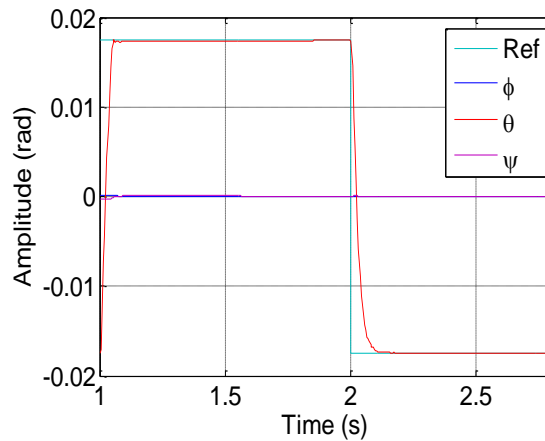
Transients' response results as shown in figure 9 and 10 are also provided to show characteristics of the system in time domain. Step inputs are fed into the system along each DOF in proper sequence. Inputs amplitude is 1 deg along rotation axes and 3 mm along translation axes respectively.



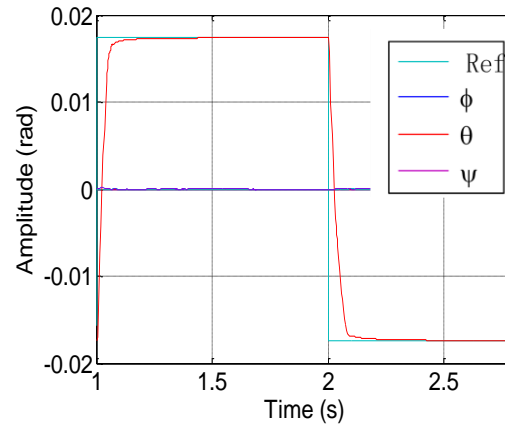
a) roll



d) roll



b) pitch



e) pitch

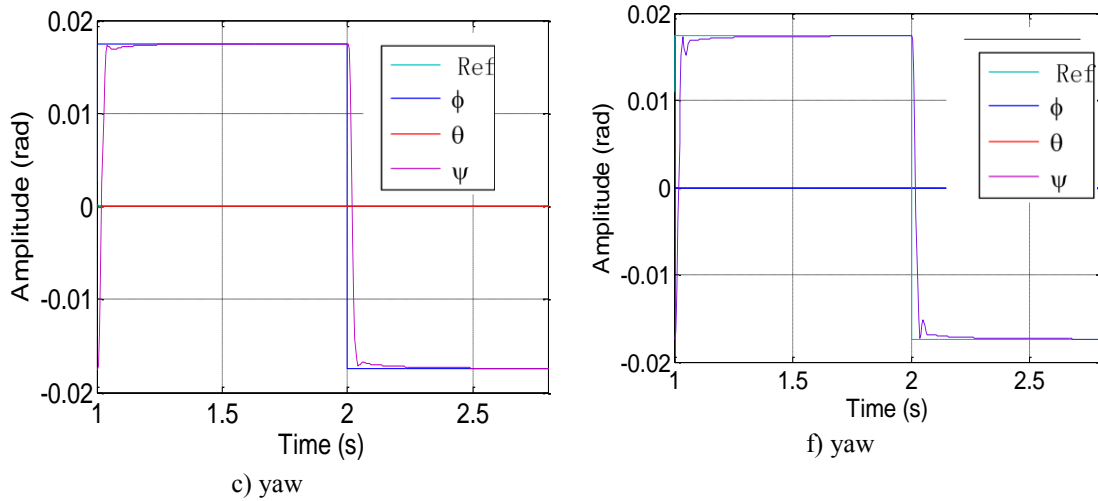
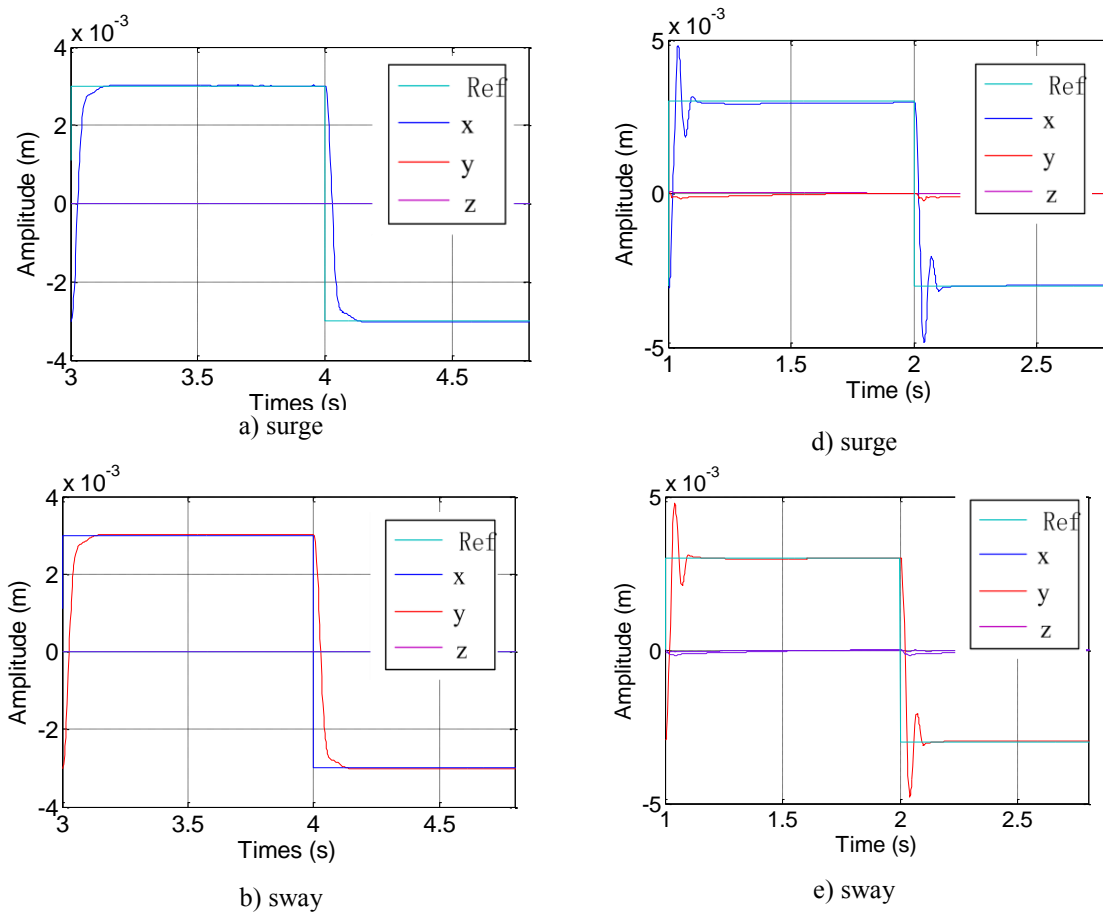


Figure 9: Simulation result of short time response with step position inputs of the modal space control (left column, a-c) and conventional control (right column, d-f)



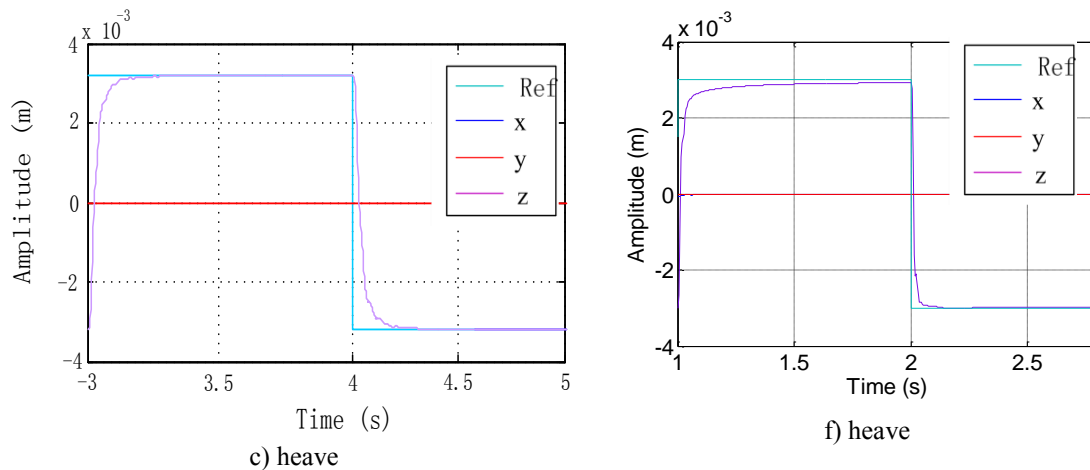


Figure 10: Simulation result of short time response with step position inputs of the modal space control (left column, a-c) and conventional control (right column, d-f)

V. CONCLUSION

Modal space decoupled control for a hydraulically driven motion simulation platform has been presented. The approach is based on singular values decomposition to the properties of joint-space inverse mass matrix, and mapping of the control and feedback variables from the joint space to the decoupling modal space. The method transformed highly coupled six-input six-output dynamics into six independent single-input single-output (SISO) 1 DOF hydraulically driven mechanical systems. It has also been shown that the methods well suited for 1 DOF hydraulically driven mechanical systems can be applied to control a multi input multi output and highly coupled hydraulically driven 6 DOF motion simulation platform.

Although, the structure of our proposed modal space controller is very similar to the conventional joint space control, the novelty in our approach is that the signals including control errors, control outputs and pressure feedbacks are transformed into decoupled modal space and also the proportional gains and dynamic pressure feedback are tuned in modal space. The results indicate that the conventional controller can only attenuate the resonance peaks of the lower eigenfrequencies of six rigid modes properly, and the peaking points of other relative higher eigenfrequencies are over damped, so the system bandwidth is limited by the lower eigenfrequencies. In addition, the results show that it is very effective to design and tune the system in modal space. The bandwidth increased substantially except surge (x) and sway (y) motions, each degree of freedom can be almost tuned independently and their bandwidths can be increased near to the undamped eigenfrequencies.

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