Dynamical System of Dengue Disease Transmission Involving the Aquatic Life Cycle

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ABSTRACT

In this article, the mathematical model for dengue disease transmission involving the aquatic life cycle was studied. Further, the equilibrium points of the mathematical model developed were determined and the stability criteria were also derived. The criteria above mentioned were dependent on the basic reproduction number which was defined as the expected value of susceptible individual got infected caused by a single infected individual. The results show that the disease-free equilibrium is locally asymptotically stable when $\mathcal{R}_0 < 1$ and the endemic equilibrium is locally asymptotically stable when $\mathcal{R}_0 > 1$. Numerical simulations are provided to show the dynamics of both human and mosquito populations upon changes of particular parameter values.

Keywords— Dengue, Aquatic, Basic reproduction number

I. INTRODUCTION

Dengue is a disease transmitted by infected Aedes aegypti mosquito through its bites carrying one of the four dengue virus serotypes called DEN-1, DEN-2, DEN-3 and DEN-4 [1, 2]. The disease transmission requires short period of time to spread and may cause death rapidly.

In this article, the transmission process was studied through mathematical models. Esteva [3] developed mathematical model of dengue transmission considering two different type of dengue viruses. Derouich *et al.* [4] formulated a mathematical model with a succession of two epidemics caused by two different viruses. Nuraini *et al.* [5] showed the internal process of dengue virus transmission in the human body. Pongsumpun [6] modeled the transmission of dengue with and without considering the extrinsic incubation period. Based on [6], Tumilaar *et al.* proposed a mathematical model of the transmission of dengue disease with intrinsic incubation, combination of intrinsic and extrinsic incubation to the dynamics of the transmission of disease dengue [7].

In this article, the mathematical model as described in [7] was modified considering the exposed stage both in human and the mosquitoes life cycle. Also, the mosquitoes aquatic life cycle was considered in this article since the process plays as important rule on the disease spread [8]. The latest component was found to the absent in the models developed in [6] and [7].

II. MATHEMATICAL MODEL

The human populations are divided into four compartments. These compartments include: susceptible, (S^h), exposed (E^h), infected, (I^h), and recovered, (R^h). The population of mosquito is divided into four compartments namely aquatic mosquitoes (A^v), susceptible (S^v), exposed (E^v), and infected (I^v). Here, we assumed that the total human populations remain constant because the birth rate equal to the mortality rate. The daily mosquito bites demoted as b was assumed not infected even from infected mosquitoes. It was also assumed that infected mosquitoes never get recovered, while the recovered humans are still posible to be susceptible and be infected.

The following are the parameters that exist in the model: N_T is the total number of human population, N_v is the total number of mosquitoes population, μ_h is human mortality rate, μ_v is the mortality rate of adult mosquito, μ_a is the average aquatic mortality rate, λ_h is the birth rate of the human population, δ_v is the average aquatic transition rate, K is the mosquito carrying capacity, k is the fraction of adult mosquito hatched rom all eggs, with $0 < p < 1$, μ_b is the average oviposition, β_{hv} is the transmission probability from infected humans to susceptible mosquitoes, β_{vh} is the transmission probability from infected mosquitoes to susceptible humans. We assume that the populations of exposed humans and mosquitoes become infectious at a rate η_h and η_v, respectively.

The compartment diagram for modifications model is shown in Figure 1.
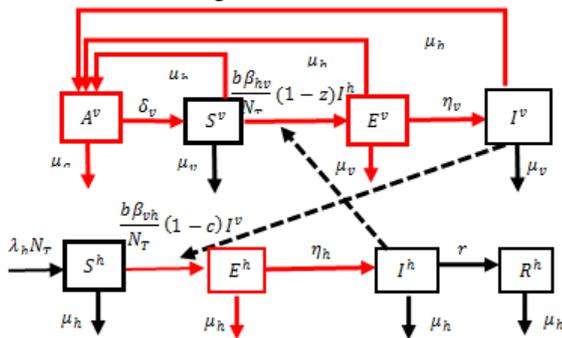

Figure 1: The diagram of modified dengue transmission model (adopted from [7] and [8]).

Model of dengue transmission in Figure 1 formulated in a system of differential equations as follows:

$$\frac{dS^h}{dt} = \lambda_h N_T - \frac{b\beta_{vh}}{N_T}(1-c)S^h I^v - \mu_h S^h,$$

$$\frac{dE^h}{dt} = \frac{b\beta_{vh}}{N_T}(1-c)S^h I^v - (\mu_h + \eta_h)E^h,$$

$$\frac{dI^h}{dt} = \eta_h E^h - (\mu_h + r)I^h,$$

$$\frac{dR^h}{dt} = rI^h - \mu_h R^h,$$

$$\frac{dA^v}{dt} = p\mu_b\left(1 - \frac{A^v}{K}\right)(S^v + E^v + I^v) - (\delta_v + \mu_a)A^v,$$

$$\frac{dS^v}{dt} = \delta_v A^v - \frac{b\beta_{hv}}{N_T}S^v(1-z)I^h - \mu_v S^v,$$

$$\frac{dE^v}{dt} = \frac{b\beta_{hv}}{N_T}S^v(1-z)I^h - (\mu_v + \eta_v)E^v,$$

$$\frac{dI^v}{dt} = \eta_v E^v - \mu_v I^v. \tag{1}$$

with
$$N_T = S^h + E^h + I^h \text{ and } N_v = S^v + E^v + I^v.$$

Furthermore, we have proven that the system (1) is positive region solution, by following Lemma 2.1.

Lemma 2.1
The set
$\Omega = \{(S^h, E^h, I^h, R^h, A^v, S^v, E^v, I^v) \in \mathbb{R}^8_+ : S^h + E^h + I^h + R^h \geq 0 \text{ and } A^v \geq 0, S^v \geq 0, E^v \geq 0, I^v \geq 0\}$ is the positive region solution.

III.   ANALYSIS MODEL

The analysis of the equilibrium points on the system (1) were obtained two types of equilibrium point, namely the disease-free equilibrium and endemic equilibrium.

3.1   The disease-free equilibrium E_1

The disease-free equilibrium E_1 of the system (1) is given by

$$E_1(S^h_*, E^h_*, I^h_*, R^h_*, A^v_*, S^v_*, E^v_*, I^v_*) =$$
$$E_1\left(\frac{N_T \lambda_h}{\mu_h}, 0, 0, 0, K\left(\frac{p\mu_b \delta_v - \mu_v \delta_v - \mu_v \mu_a}{p\mu_b \delta_v}\right),\right.$$
$$\left.K\left(\frac{p\mu_b \delta_v - \mu_v \delta_v - \mu_v \mu_a}{p\mu_b \mu_v}\right), 0, 0\right) \tag{2}$$

To analyze the stability of the equilibrium points, we need to compute the basic reproduction number of model, $\mathcal{R}_0$. Basic reproduction number, $\mathcal{R}_0$, is the expected value of susceptible individual got infected cause by a single infected individual. We calculated the basic reproduction number by using the next generation operator approach by Van Den Driessche and Watmough [9]. The next generation matrix, G, is defined as:

$$G = FV^{-1}$$

with

$$F = \begin{pmatrix} 0 & 0 & 0 & \frac{b\beta_{vh}}{N_T}(1-c)S^h \\ 0 & 0 & 0 & 0 \\ 0 & \frac{b\beta_{hv}}{N_T}(1-z)S^v & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and

$$V = \begin{pmatrix} \mu_h + \eta_h & 0 & 0 & 0 \\ -\eta_h & \mu_h + r & 0 & 0 \\ 0 & 0 & \mu_v + \eta_v & 0 \\ 0 & 0 & -\eta_v & \mu_v \end{pmatrix}$$

Thus

$$G = \begin{pmatrix} 0 & 0 & a_1 & a_2 \\ 0 & 0 & 0 & 0 \\ a_3 & a_4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

with

$a_1 = \frac{b(1-c)\beta_{vh}\eta_v\lambda_h}{\mu_h\mu_v(\eta_v+\mu_v)}$,

$a_3 = \frac{b(1-z)\beta_{hv}(-\frac{K(\delta_v+\mu_a)}{k\mu_b}+\frac{K\delta_v}{\mu_v})(\eta_h\eta_v\mu_v+\eta_h\mu_v^2)}{N_T(r+\mu_h)(\eta_h+\mu_h)\mu_v(\eta_v+\mu_v)}$,

$a_2 = \frac{b(1-c)\beta_{vh}\lambda_h}{\mu_h\mu_v}$,

$a_4 = \frac{b(1-z)\beta_{hv}(-\frac{K(\delta_v+\mu_a)}{p\mu_b}+\frac{K\delta_v}{\mu_v})}{N_T(r+\mu_h)}$.

The basic reproduction number is the largest eigenvalue of $G = FV^{-1}$, thus we get

$$\mathcal{R}_0 = \sqrt{\frac{b^2(1-c)(1-z)\beta_{hv}\beta_{vh}\eta_h\eta_v\lambda_h K\delta_v}{N_T\mu_b\mu_h(r+\mu_h)(\eta_h+\mu_h)\mu_v(\eta_v+\mu_v)\mu_v}\left(1 - \frac{1}{R_M}\right)} \tag{3}$$

or

$$\xi = \mathcal{R}_0^2 = \frac{b^2(1-c)(1-z)\beta_{hv}\beta_{vh}\eta_h\eta_v\lambda_h K\delta_v}{N_T\mu_b\mu_h(r+\mu_h)(\eta_h+\mu_h)\mu_v(\eta_v+\mu_v)\mu_v}\left(1 - \frac{1}{R_M}\right)$$

with

$$R_M = \frac{p\mu_b\delta_v}{(\delta_v+\mu_a)\mu_v} \tag{5}$$

The value of $R_M > 1$ was set as necessary condition in order to obtain the $\mathcal{R}_0$ are not imaginary value.

Theorem 3.1
The DFE E_1 of the system (1) is locally asymptotically stable if and only if $\mathcal{R}_0 < 1$.

Proof. To determine the stability of E_1, the Jacobian matrix of DFE E_1, J_{E_1} given

$$J_{E_1} = \begin{pmatrix} J_{11} & 0 & 0 & 0 & 0 & 0 & 0 & J_{18} \\ 0 & J_{22} & 0 & 0 & 0 & 0 & 0 & J_{28} \\ 0 & J_{32} & J_{33} & 0 & 0 & 0 & J_{37} & 0 \\ 0 & 0 & J_{43} & J_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & J_{55} & 0 & 0 & 0 \\ 0 & 0 & J_{63} & 0 & J_{65} & J_{66} & 0 & 0 \\ 0 & 0 & J_{73} & 0 & 0 & 0 & J_{77} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & J_{87} & J_{88} \end{pmatrix},$$

with

$J_{11} = -\mu_h$,

$J_{18} = -\frac{b(1-c)\beta_{vh}\lambda_h}{\mu_h}$,

$J_{22} = -(\mu_h + \eta_h)$,

$J_{28} = \frac{b(1-c)\beta_{vh}\lambda_h}{\mu_h}$,

$J_{32} = \eta_h$,

$J_{33} = -(\mu_h + r)$,

$J_{43} = r$,

$J_{44} = -\mu_h$,

$J_{55} = -\frac{K(\delta_v + \mu_a) + pN_v\mu_b}{K}$,

$J_{63} = \frac{bK(-1+z)\beta_{hv}(k\delta_v\mu_b - (\delta_v + \mu_a)\mu_v)}{pN_T\mu_b\mu_v}$,

$J_{65} = \delta v$,

$J_{66} = -\mu_v$,

$J_{73} = \frac{b(1-z)\beta_{hv}\left(-\frac{K(\delta_v + \mu_a)}{p\mu_b} + \frac{K\delta_v}{\mu_v}\right)}{N_T}$,

$J_{77} = -(\mu_v + \eta_v)$,

$J_{87} = \eta_v$,

$J_{88} = -\mu_v$.

and the characteristic polynomial of the matriks J_{E_1} are determined by is $\det(J_{E_1}) = 0$ is

$$(-J_{11} + \lambda)(-J_{44} + \lambda)(-J_{55} + \lambda)(-J_{66} + \lambda)(-\lambda^4 + q_1\lambda^3 + q_2\lambda^2 + q_3\lambda + q_4) = 0 \qquad (6)$$

Thus, there are eight eigenvalues and four of them are negative, that are

$\lambda_1 = J_{11} = -\mu_h$,

$\lambda_2 = J_{44} = -\mu_v$,

$\lambda_3 = J_{55} = -\frac{K(\delta_v + \mu_a) + pN_v\mu_b}{K}$,

$\lambda_4 = J_{66} = -\mu_v$.

Meanwhile the four other eigenvalues were obtained by solving equation below

$$(-\lambda^4 + q_1\lambda^3 + q_2\lambda^2 + q_3\lambda + q_4) = 0 \qquad (7)$$

where

$q_1 = -r - \eta_h - \eta_v - 2\mu_h - 2\mu_v$,

$q_2 = -(\mu_h + r)(\mu_h + \eta_h) - (\mu_h + r)(\mu_v + \eta_v) - (\mu_h + \eta_h)(\mu_v + \eta_v) - (\mu_h + r)\mu_v - (\mu_h + \eta_h)\mu_v - (\mu_v + \eta_v)\mu_v$

$q_3 = -(\mu_h + r)(\mu_h + \eta_h)(\mu_v + \eta_v) - (\mu_h + r)(\mu_h + \eta_h)\mu_v - (\mu_h + r)(\mu_v + \eta_v)\mu_v - (\mu_h + \eta_h)(\mu_v + \eta_v)\mu_v$,

$q_4 = (r + \mu_h)(\eta_h + \mu_h)(\eta_v + \mu_v)\mu_v(\xi - 1); \xi = \mathcal{R}_0^2$ \qquad (8)

The roots of the equation (7) are the other eigenvalues namely λ_5, λ_6, λ_7 and λ_8. Based on the properties of the roots of the equation (7), we gained that the roots of equation (7) satisfy the following equations [10].

$\lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 = q_1$,

$\lambda_5\lambda_6 + \lambda_5\lambda_7 + \lambda_5\lambda_8 + \lambda_6\lambda_7 + \lambda_6\lambda_8 + \lambda_7\lambda_8 = -q_2$,

$\lambda_5\lambda_6\lambda_7 + \lambda_5\lambda_6\lambda_8 + \lambda_6\lambda_7\lambda_8 = q_3$,

$\lambda_5\lambda_6\lambda_7\lambda_8 = -q_4$ \qquad (9)

As shown, $q_1 < 0$, then

$$\lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 < 0 \qquad (10)$$

This denote one of them must be negative, let $\lambda_5 < 0$. Furthermore, to check the equilibrium stability, we just need to notice the negativity of λ_6, λ_7, and λ_8.

In order to fulfill the stability criteria $\mathcal{R}_0$ was set as $\mathcal{R}_0 < 1$ which means that $\xi < 1$ (as in 4). Based on the equation (8), if $\xi < 1$ then $q_4 < 0$, we obtain

$$\lambda_5\lambda_6\lambda_7\lambda_8 > 0 \qquad (11)$$

The condition (11) is satiesfied only if $\lambda_5\lambda_6 > 0$ and $\lambda_7\lambda_8 > 0$, or $\lambda_5\lambda_6 < 0$ and $\lambda_7\lambda_8 < 0$.

Because $\lambda_5 < 0$ then λ_6 that satisfy the conditions (9) is $\lambda_6 < 0$.

As shown, $q_3 < 0$, then we obtain

$$\lambda_5\lambda_6\lambda_7 + \lambda_5\lambda_6\lambda_8 + \lambda_6\lambda_7\lambda_8 < 0$$

or

$$\lambda_5\lambda_6(\lambda_7 + \lambda_8) + \lambda_6\lambda_7\lambda_8 < 0. \qquad (12)$$

The inequality (12) is satisfied if $\lambda_5\lambda_6(\lambda_7 + \lambda_8) < 0$ and $\lambda_6\lambda_7\lambda_8 < 0$.

As assumed before, $\lambda_5 < 0$. Also, it was showed that $\lambda_6 < 0$. Thus, we get

$$(\lambda_7 + \lambda_8) < 0 \text{ and } \lambda_7\lambda_8 > 0 \qquad (13)$$

The condition (13) can be satisfied if and only if $\lambda_7 < 0$ and $\lambda_8 < 0$. Thus, we know that all of the eigenvalue are negative. Therefore, if $\mathcal{R}_0 < 1$, then the disease-free equilibrium E_1 of the system (1) is locally asymptotically stable.

∎

3.2 The Endemic Equilibrium

The endemic equilibrium of the system (1) is given by

$E_2(\,A_{**}^v, S_{**}^v, E_{**}^v, I_{**}^v, S_{**}^h, E_{**}^h, I_{**}^h, R_{**}^h)$

with

$$S_{**}^h = \frac{N_T^2 \lambda_h}{b(1-c)I^v \beta_{vh} + N_T \mu_h},$$
$$E_{**}^h = \frac{b(1-c)I^v S^h \beta_{vh}}{N_T(\eta_h + \mu_h)},$$
$$I_{**}^h = \frac{E^h \eta_h}{r + \mu_h},$$
$$R_{**}^h = \frac{I^h r}{\mu_h},$$
$$A_{**}^v = \frac{pK(N_v)\mu_b}{K(\delta_v + \mu_a) + k(N_v)\mu_b},$$
$$S_{**}^v = \frac{A^v N_T \delta_v}{bI^h(1-z)\beta_{hv} + N_T \mu_v},$$
$$E_{**}^v = \frac{bI^h S^v(1-z)\beta_{hv}}{N_T(\eta_v + \mu_v)},$$
$$I_{**}^v = \frac{E^v \eta_v}{\mu_v}.$$

(14)

Theorem 3.2

If $\mathcal{R}_0 > 1$, then endemic equilibrium E_2 is locally asymptotically stable.

Proof. The Jacobian matrix at E_2 of the system (1) is given by

$$J_{E_2} = \begin{pmatrix} a_{11} & 0 & 0 & 0 & 0 & 0 & 0 & a_{18} \\ a_{21} & a_{22} & 0 & 0 & 0 & 0 & 0 & a_{28} \\ 0 & a_{32} & a_{33} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{43} & a_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & 0 & 0 & 0 \\ 0 & 0 & a_{63} & 0 & a_{65} & a_{66} & 0 & 0 \\ 0 & 0 & a_{73} & 0 & 0 & a_{76} & a_{77} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_{87} & a_{88} \end{pmatrix}$$

The characteristic polynomial of the matriks J_{E_2} are determined by $\det(J_{E_2}) = 0$ is

$(a_{44} - \lambda)(a_{55} - \lambda)\Big((a_{11} - \lambda)(a_{22} - \lambda)(a_{33} - \lambda)(a_{66} - \lambda)(a_{77} - \lambda)(a_{88} - \lambda) + a_{32}a_{87}(a_{66}a_{73} - a_{63}a_{76} - a_{73}\lambda)\big(a_{18}a_{21} + a_{28}(-a_{11} + \lambda)\big)\Big) = 0.$

The two eigenvalues are obtained $\lambda_1 = a_{44} = -\mu_h$, $\lambda_2 = a_{55} = -\mu_v$, and the six other eigenvalues were obtained by solving characteristics equations following:

$(\lambda^6 + p_1 \lambda^5 + p_2 \lambda^4 + p_3 \lambda^3 + p_4 \lambda^2 + p_5 \lambda + p_6) = 0$

Based on the Routh-Hurwitz criterion [11], the equation (15) of the endemic equilibrium E_2 is stable if fulfill the stability criterion below

$p_1 > 0, p_2 > 0, p_3 > 0, p_4 > 0, p_5 > 0, p_6 > 0$ and

$p_1 p_2 p_3 p_4 p_5 p_6 - p_3^2 p_4 p_5 p_6 - p_1^2 p_4^2 p_5 p_6 - p_1 p_2^2 p_5^2 p_6 + p_2 p_3 p_5^2 p_6 + 2p_1 p_4 p_5^2 p_6 - p_5^3 p_6 - p_1 p_2 p_3^2 p_6^2 + p_3^3 p_6^2 + p_1^2 p_3 p_4 p_6^2 + 2p_1^2 p_2 p_5 p_6^2 - 3p_1 p_3 p_5 p_6^2 - p_1^3 p_6^3 > 0$
(16)

Noted that if $\mathcal{R}_0 > 1$, then $p_1 > 0$, $p_2 > 0$, $p_3 > 0$, $p_4 > 0$, $p_5 > 0$, and $p_6 > 0$. Further, $p_1 p_2 p_3 p_4 p_5 p_6 - p_3^2 p_4 p_5 p_6 - p_1^2 p_4^2 p_5 p_6 - p_1 p_2^2 p_5^2 p_6 + p_2 p_3 p_5^2 p_6 + 2p_1 p_4 p_5^2 p_6 - p_5^3 p_6 - p_1 p_2 p_3^2 p_6^2 + p_3^3 p_6^2 + p_1^2 p_3 p_4 p_6^2 + 2p_1^2 p_2 p_5 p_6^2 - 3p_1 p_3 p_5 p_6^2 - p_1^3 p_6^3 > 0$ if $\mathcal{R}_0 > 1$. Thus, if $\mathcal{R}_0 > 1$ then the condition (16) is satiesfied. Therefore, by Routh-Hurwitz criterion the endemic equilibrium E_2 for system (1) is locally asymtotically stable if $\mathcal{R}_0 > 1$. ∎
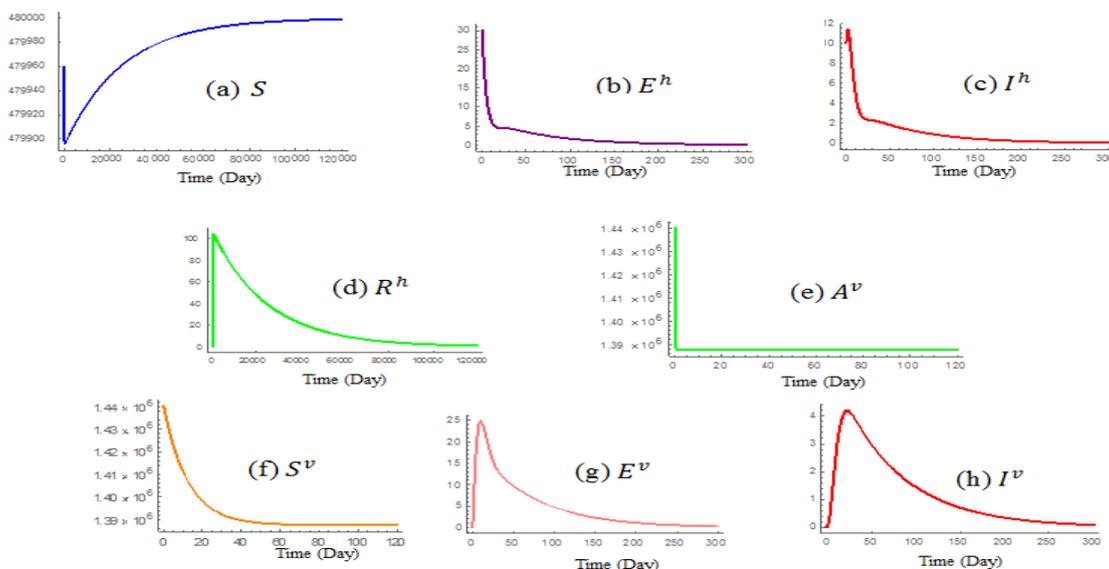

Figure 2: Dynamics of human populations and the population dynamics of mosquitoes for the disease-free equilibrium

IV.   NUMERICAL SIMULATION

Simulations were to justify the stability properties of the equilibrium points based on the theorem in section 3 and to see the influence of parameter variations in dynamics system. The initial value of $S^h(0) = 479960$, $E^h(0) = 30$, $I^h(0) = 10$,

$R^h(0) = 0, \quad A^v(0) = 1440000, \quad S^v(0) = 1440000,$
$E^v(0) = 0$ and $I^v(0) = 0$.

4.1 The population dynamics of human and mosquito for the disease-free equilibrium

The parameters used in this simulation for the disease-free equilibrium were $\mu_b = 11.2$, $\mu_v = 0.08$, $\mu_a = \frac{1}{4}$, $\delta_v = 0.08$, $\eta_v = 0.02$, $\mu_h = 0.00004$, $\eta_h = 0.17$, $r = \frac{1}{3}$, $p = 0.78$, $\lambda_h = 0.00004$, $\beta_{hv} = 0.21$, $\beta_{vh} = 0.21$, $K = 1440000$, $b = 0.6$, $c = 0.2$, $z = 0.3$ (taken from [8] and [12]). Based on the parameters, we acquired the basic reproduction number $\mathcal{R}_0 = 0.438649 < 1$ and the disease free equilibrium $E_1(S_*^h, E_*^h, I_*^h, R_*^h, A_*^v, S_*^v, E_*^v, I_*^v) = E_1(480000, 0, 0, 0, 1385604.39, 1385604.39, 0, 0)$.

The simulation results for the human population and the mosquito population for the disease-free equilibrium.

In Figure 2a it can be seen that the susceptible humans population rapidly decrease in the begining of time simulation, then the population increase with time and finally approaching the disease-free equilibrium. The exposed humans population rapidly decrease in the begining of time simulation, and continuously decrease until to the end of simulation time (Figure 2b). The infected humans and the recovery humans population, rapidly increase in the begining at time simulation, then the population decrease with time and finally approaching the disease-free equilibrium (Figure 2c-2d).

The mosquitoes aquatic population in short time decrease in the begining at time simulation and then approaching the disease-free equilibrium (Figure 2e). The susceptible mosquitoes initially increase in the begining at time simulation and then approaching the disease-free equilibrium (Figure 2f). The exposed mosquitoes and infected mosquitoes population initially increase, but the population increase with time and finally approaching the disease-free equilibrium (Figure 2g-2h). The results are consistent with Theorem 3.1 that the disease-free equilibrium
$E_1(S_*^h, E_*^h, I_*^h, R_*^h, A_*^v, S_*^v, E_*^v, I_*^v)$
$= E_1(480000, 0, 0, 0, 1385604.39, 1385604.39, 0, 0)$
is locally asymptotic stable if $\mathcal{R}_0 < 1$.

4.2 The population dynamics of human and mosquito for the endemic equilibrium

The parameters used in this simulation for the endemic equilibrium were $\mu_b = 11.2$, $\mu_v = 0.08$, $\mu_a = \frac{1}{4}$, $\delta_v = 0.08$, $\eta_v = 0.2$, $\mu_h = 0.00004$, $\eta_h = 0.17$, $r = \frac{1}{3}$, $p = 0.78$, $\lambda_h = 0.00004$, $\beta_{hv} = 0.21$, $\beta_{vh} = 0.21$, $K = 1440000$, $b = 0.8$, $c = 0.2$, $z = 0.3$ (taken from [8] and [12]). Based on the parameters then acquired the basic reproduction number $\mathcal{R}_0 = 1.10529 > 1$ and the point remains endemic
$E_2(S_{**}^h, E_{**}^h, I_{**}^h, R_{**}^h, A_{**}^v, S_{**}^v, E_{**}^v, I_{**}^v) =$
$E_2(392918, 20.4851, 10.4462, 87051.4, 1385604.39,$
The simulation results for the human and mosquito populations for the endemic equilibrium are shown in Figure 3.
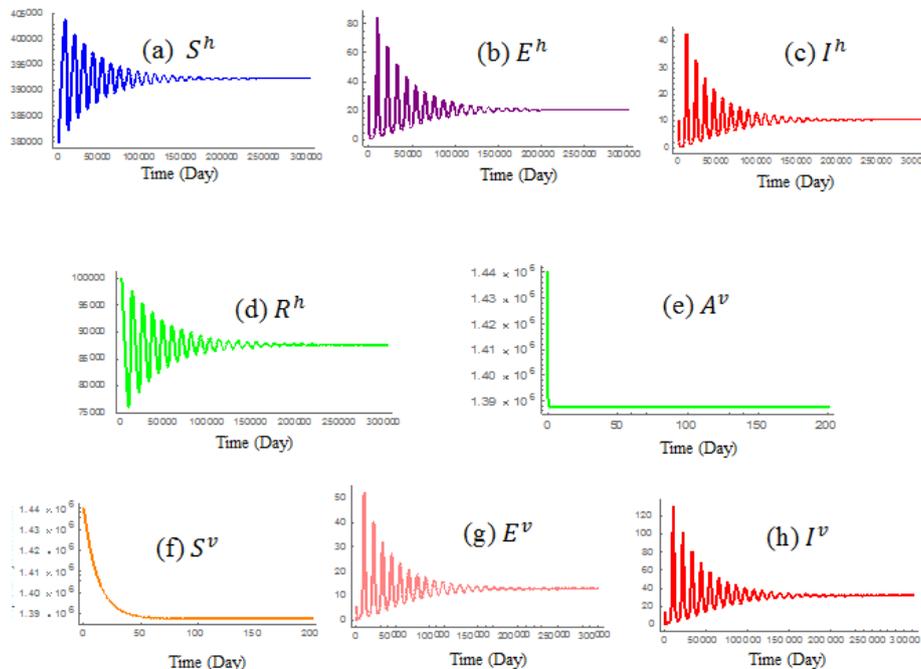

Figure 3: The dynamics of human populations and the population dynamics of mosquitoes for the endemic equilibrium

The susceptible humans population initially increase in short time, then occur fluctuations and finally approaching the endemic equilibrium (Figure 3a). The exposed humans population initially decrease but after certain time occur fluctuations, then approaching the endemic equilibrium (Figure 3b). Similarly occur to the infected humans population and the recovered humans

population, the two populations occur fluctuations, then approaching the endemic equilibrium (Figure 3c-3d).

The mosquitoes aquatic population in the beginning of simulations decrease until approaching the endemic equilibrium (Figure 3e). Similarly occur to the susceptible mosquitoes population, the susceptible mosquitoes population initially decrease, then approaching the endemic equilibrium (Figure 3f). While the infected mosquitoes and exposed mosquitoes population, initially increased and occur fluctuations, then finally approaching the endemic equilibrium (Figure 3g-3f). The results are consistent with Theorem 3.1 that the endemic equilibrium

$$E_2(S_{**}^h, E_{**}^h, I_{**}^h, R_{**}^h, A_{**}^v, S_{**}^v, E_{**}^v, I_{**}^v) =$$
$$E_2(392918, 20.4851, 10.4462, 87051.4, 1385604.39,$$
$$1385560.07, 12.6646, 31.6614)$$

is locally asymptotic stable if $\mathcal{R}_0 > 1$.

4.3 The influence of the number of eggs produced from each compartment per capita (μ_b)

There are three variations of parameter μ_b was observed, taken from interval [0-11.2] from [8], and the values of other parameters for the diesease-free equilbrium and the endemic equilibrium is fixed. Figure 4 and 5 shows the effects that occur if the number of eggs produced decreased.

Figure 4:   The effect of μ_b to the infected humans population, the mosquitoes aquatic and the infected mosquitoes for the diesease-free equilibrium
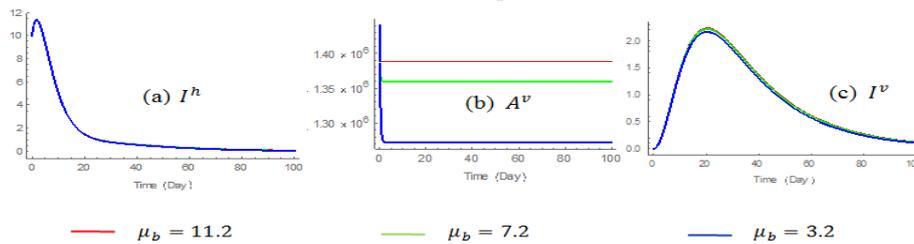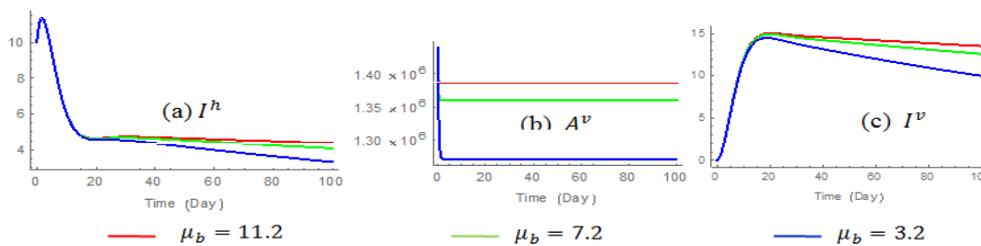

Figure 5:   The effect of μ_b to the infected humans population, the mosquitoes aquatic and the infected mosquitoes for the endemic equilibrium

Based on Figure 4a and 5a we can see that decreasing of the number of eggs produced causes the number of the infected humans is on the wane. The same thing occurs to the mosquitoes aquatic (Figure 4b and 5b) and the infected mosquitoes (Figure 4c and 5c). This implies that decreasing of the number of eggs produced helping reduce the rate of spread of dengue disease.

4.4 Effect of the mortality rate from mosquitoes aquatic (μ_a)

There are three variations of parameter μ_a was observed taken from interval [0.01-0.47] from [8], and the values of other parameters for the diesease-free equilbrium and the endemic equilibrium is fixed. Figure 6 and 7 shows the effects that occur if the mortality rate of mosquitoes increased.

Figure 6:   The effect of μ_a to the infected humans population, the mosquitoes aquatic and the infected mosquitoes for the desease-free equilibrium
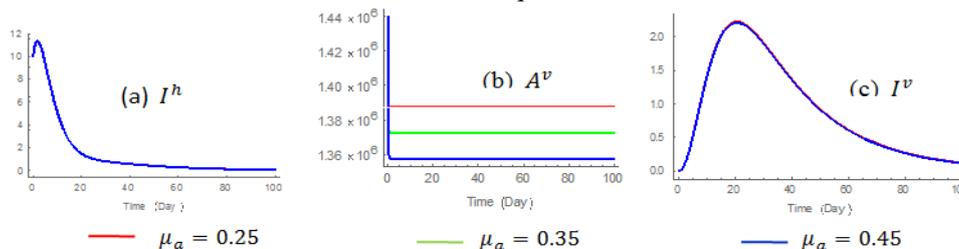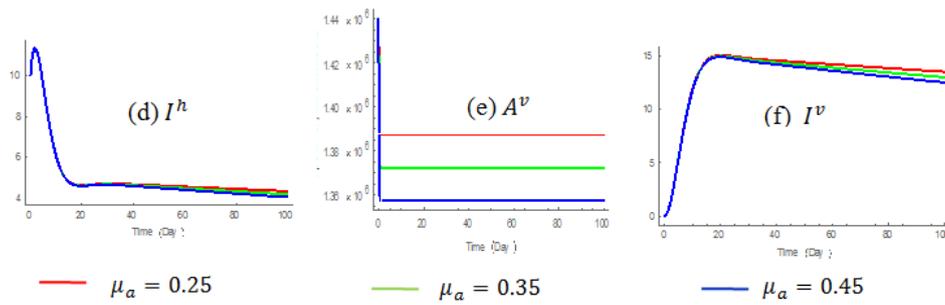

Figure 7:   The effect of μ_a to the infected humans population, the mosquitoes aquatic and the infected mosquitoes for the endemic equilibrium

(d) I^h — (e) A^v — (f) I^v

$\mu_a = 0.25$ $\mu_a = 0.35$ $\mu_a = 0.45$

Based on Figure 6a we can see that increasing of the mortality rate from mosquitoes aquatic causes the number of the infected humans for the diesease-free equilibrium is not too different. While the number of the infected humans for the endemic equilibrium occur change the number of population is decreasing (Figure 7a). The mosquitoes aquatic (Figure 6b and 7b) and the infected mosquitoes (Figure 6c and 7c) population is decreasing. This implies that increasing of the mortality rate from mosquitoes aquatic helping reduce the rate of spread of dengue disease.

V.   CONCLUSION

The mathematical model involving the transmission of dengue disease with aquatic life cycles was considered to describe the transmission of dengue disease. The mathematical model of dengue disease involving aquatic life cycle has two equilibrium points, then namely the disease-free equilibrium and the endemic equilibrium. If $\mathcal{R}_0 < 1$, the disease-free equilibrium E_1 is locally asymptotically stable. The endemic equilibrium E_2 is locally asymptotically stable if $\mathcal{R}_0 > 1$. The decrease of μ_b and the increase of μ_a can help reduce the rate of disease transmission in the population so that there is no outbreak in the population.

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