

# Electromagnetic Beam Causing Instability

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## ABSTRACT

In laser, a perturbation in the laser intensity has a tendency to grow. The regions where of intensity is higher, hence the electron density falls and gives rise to a higher refractive index. This region attracts power from neighborhood causing growth of intensity perturbation. Hence we get filaments.

**Keywords**— Electromagnetic, Beam, Predominate

## I. INTRODUCTION

In plasmas, where relativistic mass and density nonlinearities predominate, one observes frequency downshift and leads to relativistic self-focusing of the laser beam [1-3]. Paraxial ray theory [4-5] of relativistic and ponderomotive self-focusing has been developed. Fedosejevs *et al* [6] have reported experiments on many gases including helium, hydrogen and nitrogen etc. They employed a  $0.3TW, 250 - fs$  laser pulse focused to an intensity of  $3 \times 10^{17} W/cm^2$ . They observed relativistic self-focusing in high density hydrogen gas.

Demchenko and Hussein [6] have developed the theory of transverse wave self-focusing in anisotropic plasma.. Fedosejevs *et al* [7] have studied the relativistic self-focusing in high-density gas jet targets. Couairon and

$$\mathbf{E} = \hat{y}[\vec{E}_0 + E_1(\vec{r})] \exp[-i(\omega t - kz)], \quad (1)$$

where  $\vec{E}_0$  is a constant and  $E_1(\vec{r})$  is the perturbation. The perturbed intensity is  $I = I_0 + \Delta I$ , where  $I_0 = \eta c E_0^2 / 8\pi$ ,  $\Delta I = \eta c E_0 (E_1 + E_1^*) / 8\pi$  and  $\eta$  is the refractive index of the semiconductor. One may note that  $\Delta I$  is a function of  $\vec{r}$ . Let the perturbation in the electron and hole densities be

$$n_e = n_{e0} + \Delta n,$$

$$n_h = n_{h0} + \Delta n,$$

where changes in electron and hole densities are taken equal as these carriers are produced simultaneously via electron hole-pair production. The perturbed electron density is governed by

$$\frac{dn}{dt} = W_{21} - W_o,$$

$$\frac{d}{dt}(\Delta n) = \alpha [f_c(\epsilon_2) - f_v(\epsilon_1)] n_o \Delta I + \alpha \frac{\partial}{\partial n_o} [f_c(\epsilon_2) - f_v(\epsilon_1)] \Delta n I_o. \quad (2)$$

Berge [8] also studied filamentation of ultra-short pulses in ionizing media. Liu and Tripathi [9] have given an elegant survey of nonlinear interaction of electromagnetic waves with electron beams and plasmas. Studies on self-focusing have also been extended to parabolic and non parabolic semiconductors where nonlinearity arises due to Ohmic heating and redistribution of free carriers and due to nonparability of energy bands.

In this article, we study the instability of an infrared laser in a semiconductor under population inversion. The initial wave front is plane and initial intensity distribution is uniform. When the intensity of the beam is perturbed, it causes spatial modulation in the dielectric constant. The regions of higher intensity cause higher stimulated emission via electron-hole recombination, hence electron density falls over there giving rise to higher refractive index. Thus the high intensity regions attract more power from the surrounding portion of the beam leading to the growth of the perturbation. In section 2, we carry out the instability analysis.

## II. INSTABILITY ANALYSIS

When a small perturbation is given to the equilibrium in the field of the laser

The first term on RHS represents increase in the rate of electron production due to the increase in the local laser intensity. The second term represents the increase in the rate of electron production due to the increase in

density. Here we solve the spatial problem, hence we take  $d(\Delta n)/dt = 0$ ,

$$\Delta n = \frac{-[f_c(\epsilon_2) - f_v(\epsilon_1)]_{n_o} \Delta I}{\frac{\partial}{\partial n_o} [f_c(\epsilon_2) - f_v(\epsilon_1)] I_o} \tag{3}$$

One may note that  $\Delta I$  is a function of  $x, y, z$ , hence  $\Delta n$  is spatially nonuniform.

The effective permittivity of the active region is

$$\epsilon = \epsilon_L - \frac{(\omega_{pe}^2 + \omega_{ph}^2)}{\omega^2},$$

$$\epsilon = \epsilon_o + \epsilon_2 E_o (E_1 + E_1^*), \tag{4}$$

where

$$\epsilon_2 = \frac{\omega_p^2}{\omega^2} \left( 1 + \frac{m_e}{m_h} \right) \frac{[f_c(\epsilon_2) - f_v(\epsilon_1)]}{n_o \frac{\partial}{\partial n_o} [f_c(\epsilon_2) - f_v(\epsilon_1)]} \frac{1}{E_o^2}. \tag{5}$$

Following Liu and Tripathi, we deduce the growth rate and the optimum size of maximally growing intensity perturbation.

The wave equation for the propagation of the em wave is

$$\nabla^2 \vec{E} + \nabla \left( \frac{\vec{E} \cdot \nabla \epsilon}{\epsilon} \right) + \frac{\omega^2}{c^2} \epsilon \vec{E} = 0 \tag{6}$$

where we have used  $\nabla \cdot \vec{D} = 0$  substituting for  $\vec{E}$  from Eq.(3) and taking  $k_o = \omega/c \in^{1/2}$ , we obtain the equation for  $E_1$ .

$$\nabla^2 E_1 - 2ik_o \frac{\partial E_1}{\partial z} + \frac{\epsilon_2 E_o^2}{\epsilon_o (E_o^2)} \nabla \left( \frac{\partial}{\partial y} (E_1 + E_1^*) \right) + \frac{\omega^2 \epsilon_2 E_o (E_1 + E_1^*)}{c^2} = 0, \tag{7}$$

On expressing  $E_1 = E_{1r} + iE_{1i}$  and separating real and imaginary parts, we get

$$\nabla^2 E_{1r} + 2k_o \frac{\partial}{\partial z} E_{1i} + \frac{2\epsilon_2 E_o^2}{\epsilon_o (E_o^2)} \frac{\partial^2}{\partial y^2} E_{1r} + \frac{2\omega^2}{c^2} \epsilon_2 E_o^2 E_{1r} = 0, \tag{8}$$

and

$$\nabla^2 E_{1i} - 2k_o \frac{\partial}{\partial z} E_{1r} = 0, \tag{9}$$

In order to solve these coupled equations. For  $E_{1r}$  and  $E_{1i}$ , we employ a complex notation and assume variation of the form  $\exp[-i(k_{||} z + k_{\perp} r_{\perp})]$ , where  $\vec{r}_{\perp} = x\hat{i} + y\hat{j}$

Now using above expressions in Eqs.(8) and (9), we get two equations. These equations. have non-trivial solution only when determinant of coefficients of  $E_{1r}$  and  $E_{1i}$  is zero i.e. when

$$(k_{\perp}^2 + k_{||}^2) [(k_{||}^2 + k_{\perp}^2) + \frac{2\epsilon_2 E_o^2}{\epsilon_o (E_o^2)} k_y^2 - \frac{2\omega^2}{c^2} \epsilon_2 E_o^2] = 4k_o^2 k_{||}^2 \tag{10}$$

Taking  $k_{\parallel}^2 < k_{\perp}^2$  which is a valid assumption, Eq.(10) simplifies to

$$k_{\parallel}^2 = \frac{k_{\perp}^2}{4k_o^2} \left[ k_{\perp}^2 + \frac{2\epsilon_2 E_o^2}{\epsilon_o (E_o^2)} k_y^2 - \frac{2k_o^2 \epsilon_2 E_o^2}{\epsilon_o (E_o^2)} \right] \tag{11}$$

when

$$\frac{k_o^2 \epsilon_2 E_o^2}{\epsilon_o (E_o^2)} > \frac{k_{\perp}^2}{2} + \frac{\epsilon_2 E_o^2}{\epsilon_o (E_o^2)} k_y^2,$$

$k_{\parallel}$  becomes imaginary i.e. perturbation grows as it advances in the  $z$  direction. The growth rate is given by

$$\Gamma = ik_{\parallel} = \frac{k_{\perp}}{2k_o} \left[ -k_{\perp}^2 - \frac{2\epsilon_2 E_o^2}{\epsilon_o (E_o^2)} k_y^2 + \frac{2k_o^2 \epsilon_2 E_o^2}{\epsilon_o (E_o^2)} \right]^{1/2} \tag{12}$$

To discuss the behavior of the growth rate, we consider a special case consider  $k_y = 0, k_{\perp} = k_x$ . In this case, wave vector of perturbation is at right angle to

the electric wave and perturbation propagates as TE wave. The growth rate from equation (14) comes out to be

$$\Gamma = \frac{k_x}{k_o} \left[ 2k_o^2 \frac{\epsilon_2 E_o^2}{\epsilon_o (E_o^2)} - k_x^2 \right]^{1/2}.$$

$\Gamma$  is a function of  $k_x$  and shows a maximum at  $k_x$  corresponding to  $d\Gamma/dk_x = 0$ . A graph is plotted both  $\Gamma/k_o$  and  $k_x/k_o$  (cf. Fig.3) for value of  $\epsilon_2 E_o^2/\epsilon_o = 0.4$ .

### III. DISCUSSION

Stimulated electron hole recombination offers an efficient mechanism of self-focusing and filamentation instability. A perturbation in the laser intensity causes more intensity where intensity is large due to stimulated recombination of electrons and holes and lesser intensity

where intensity is low. This instability grows with time, we get high intensity filaments. The growth rate varies linearly with laser intensity and carrier concentration.

The consequence of this instability is that as a plane wave propagates in a non-linear medium it splits up into filaments in the direction normal to its propagation.

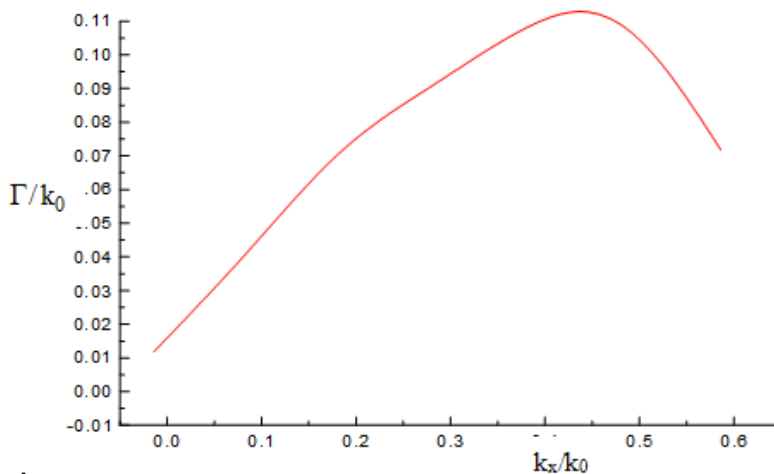


Figure 1: Variation of Growth rate of filamentation ( $\Gamma/k_o$ ) vs. ( $k_x/k_o$ ) for  $\epsilon_2 E_o^2/\epsilon_o = 0.4$

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