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Contra C*-Continuity in Topological Spaces

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ABSTRACT

In this paper, we introduce the notions of contra C*-continuity and contra C(S)-continuity in topological spaces and study the relations of contra C*-continuity with various generalized contra continuity maps.

Keywords-- Contra C* -Continuity, C* -Continuity, C(S)-Continuity, Contra C(S)-Continuity

I. INTRODUCTION AND PRELIMINARIES

Throughout this paper, X and Y denote topological spaces (X, τ) and (Y, σ) respectively.For a subset A of X , the closure, the interior and the complement of A are denoted by cl(A) , int(A) and A^C respectively.

Definition 1.1. A subset S of X is called

a) an A-set if $S=G \cap F$ where G is open and F is regular closed in X,
b) a t-set if int(S)=int(cl(S)),
c) a B-set if $S= G \cap F$ where G is open and F is a t-set in X,
d) an α*-set if int(S)=int(cl(int(S))),
e) a C-set (due to Sundaram) if $S= G \cap F$ where G is g-open and F is a t-set in X,
f) a C-set (due to Hatir,Noiri, and Yuksel) if $S= G \cap F$ where G is open and F is an α*-set in X.

Proposition 1.2 : Every C(S) -set in X is a C*-set in X.

Proof : Let S be a C(S) -set in X. Then $S = A \cap B$ Where A is g-open and B is a t-set in X.

Proposition 1.3 : Every C-set in X is a C*-set in X.

Proof : Let S be a C-set set in X. Then $S = A \cap B$ Where A is open and B is an α*-set in X. Since every open set in g-open, we see that S is a C*-set in X.

II. CONTRA C*-CONTINUITY

Definition 2.1 : A map $f : X \rightarrow Y$ is said to be contra C*-continuous if $f^{-1}(F)$ is a C*-set in X for every closed set F in Y.

Definition 2.2 : A map $f : X \rightarrow Y$ is said to be contra C(S) -continuous if $f^{-1}(F)$ is a C(S)-set in X for every closed set F in Y.

Definition 2.3 [4] : A map $f : X \rightarrow Y$ is said to be contra gp- continuous if $f^{-1}(F)$ is a gp- open in X for every closed set F in Y.

Proposition 2.4 : Every contra C(S)-continuous map is contra C*-continuous

Proof . Let $f : X \rightarrow Y$ be a contra C(S) continuous map. Since every C(S)-set in X is a C*-set in X, f is contra C*-continuous.

Example 2.5 : Let X = { a, b, c } Y = {x,y}, $\tau = \{\phi, \{a,b\}, X\}$ and $\sigma = \{\phi, \{x\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = x and f(b) = f(c) = y. Here, $f^{-1}(\{y\}) = \{b,c\}$ is a C*-set but not a C(S)-set in X. Thus, f is contra C*-continuous but not contra C(S)-continuous.

Remark 2.6 : (a) contra C* -continuity and C* -continuity are independent.
(b) contra C(S) -continuity and C(S) -continuity are independent
(c) contra C*-continuity and contra gp -continuity are independent
(d) contra C(S)-continuity and contra gp-continuity and
(e) contragp -continuity and gp -continuity
are independent as seen from the following examples.

Example 2.7 : Let X = Y = { a, b, c } $\tau = \{\phi, \{a\}, \{a,b\}, X\}$ and $\sigma = \{\phi, \{x\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = b, f(b) = a and f(c) = c. Here $f^{-1}(\{a\}) = b$ is a C*-set and a C(S)-set in (X, τ). Thus f is C*-continuous and C(S)-continuous but is neither contra C*-continuous nor contra C(S)-continuous .

Example 2.8 : Let (X, τ) be as in example 1.7. Let Y = {x,y} and $\sigma = \{\phi, \{x\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) =f(c) = x and f(b) = y. Then f is contra C*-continuous and contra C(S)-continuous but is neither C*-continuous nor C(S)-continuous .

Example 2.9 : Let (X, τ) be as in example 1.7. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = a, f(b) = c and f(c) = b. Then f is gp-continuous and since $f^{-1}(\{b,c\}) = \{b, c\}$ is not gp-open in (X, τ), f is not contra gp-continuous. Also, define

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g : (X, τ) → (X, τ) by g(a) = c, g(b) =b g(c) = a. Then g is contra gp-continuous and but not gp-continuous.

Example 2.10 : Let (X, τ) be as in example 1.7. Let Y = {x,y} and σ = {ϕ, {x}, Y}. Define f : (X, τ) → (Y, σ) by f(a) = f(c) = y and f(b) =x. Then f is contra gp-continuous but is neither contra C*-continuous nor contra C(S)-continuous , for f^{-1}({y}) = {a,c} is a gp-open but is neither a C*-set nor a C(S)-set in (X, τ).

Example 2.11 : Let f : (X, τ) → (X, τ) be as defined in example 1.9. Then f is contra C*-continuus and contra C(S)-continuous but not contra gp-continuous.

Example 2.12 : A map f : X →Y is contra g-continuous (resp, contra rg-continuous, contra αg-continuous, contra gα**-continuous) if and only if f^{-1}(F) is g-open (resp, rg-open, αg open, gα**-open) in X for every closed set F in Y.

Proof . The proof follows from the result : f^{-1}(Y-A) = X - f^{-1}(A) for any subset A of Y.

Proposition 2.13 : Let X →Y be contra g-continuous. Then

a) f is contra C*-continuous
b) f is contra C(S)-continuous
c) f is contra αg-continuous
d) f is contra gp-continuous

Proof . Since every g-open set in X is a C*-set, a C(S)-set αg-open and gp-open in X, the proof follows easily.

However, the converses need not be true as seen from the following examples.

Example 2.14 : Let X={a,b,c}, τ = {ϕ, {a},{a,b} X}. Define f : (X, τ) → (Y, σ) by
f(a) = a, f(b) = c and f(c) = b. Then f is contra C*-continuous and
contra C(S)- continuous but not contra g-continuous.

Example 2.15 : Let X = { a, b, c } Y = {x,y}, τ = {ϕ, {a}, {a,b}, X} and σ = {ϕ, {x}, Y}. Define f : (X, τ) → (Y, σ) by f(a) = f(c) = y and f(b) = x . Here f^{-1}({y}) = {a,c} is αg open, gp-open but not g-open in (X, τ).Thus f is contra αg-continuous and
contra gp-continuous but not contra g-continuous.

Proposition 2.16 : A contra αg-continuous map is contra gp-continuous .

Proof. Since every αg-open set is gp-open, the proof follows.

However, the converse need not be true as seen from the following examples;

Example 2.17 : Let X = {ϕ,a, b}, X } Y = {x,y}, τ = {ϕ, {a,b}, X} and σ = {ϕ, {x}, Y}. Define f : (X, τ) → (Y, σ) by f(a) = x and f(b) = f(c)=y. Here, f^{-1}({y}) = {b,c} is gp-open but not αg-open in : (X, τ). Thus f is contra gp-continuous but not
contra αg-continuous.

Remark 2.18 : Contra αg-continuous is independent of contra C*-continuity and contra C(S)-continuity as seen from the following examples ;

Example 2.19 : Let X = { a, b, c } Y = {x,y}, τ = {ϕ, {a}, {a,b}, X} and σ = {ϕ, {x}, Y}. Define f : (X, τ) → (Y, σ) by f(a) = f(c) = y and f(b) = x . Then f is contra αg-continuous , but is neither contra C(S)-continuous nor contra C*-continuous. Also define g : (X, τ) → (X, τ) by g(a) =a, g(b) = c and g(c) = b. Here g^{-1}({c}) = {b} and g^{-1}({b, c}) = {b,c} are both C*-sets and C(S)-sets but {b,c} is not αg open (X, τ).Thus g is contra C*-continuous and contra C(S)-continuous but not contra αg-continuous.

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